Flexible Greenhouse Gas Emission Banking Systems

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Summary of Findings

This final report reports on the value of time flexibility in greenhouse gas permit trading systems, from the application of non-unitary intertemporal trading rates for permit borrowing and banking. Permit systems that allow banking and borrowing are seeing growing regulatory interest both nationally and internationally. The sulfur dioxide trading program, authorized by the 1990 Clean Air Act Amendments, is the best known and most extensive venture in marketable permits to date. This program allows firms to bank, but not borrow, permits. Another domestic example is the current corporate average fuel economy (CAFE) regulations that allow automobile manufacturers to bank *and borrow* fuel economy credits for up to three years (49 USC 32903). Certainly, however, the grandest use yet envisioned for marketable permits was contained in a recent draft proposal by the U.S. Department of State to allow nations of the world to trade, bank and borrow greenhouse gas permits under the Framework Convention on Climate Change (USDOS, 1997).

Despite the reluctance of developing nations to allow any form of emission trading, the Kyoto Protocol signed last year does allow emission trading among Annex B (developed) nations (United Nation, Article 16bis 1997). The details on emission trading are still being negotiated. Whether or not the banking and borrowing, in particular, of greenhouse gas permits will be allowed is not yet determined. A carefully structured trading system that makes some provision for permit banking and borrowing has significant merit. It can maintain market incentives for efficient emission reductions while still providing individual parties the time-flexibility that they may need to meet their negotiated obligations.

This research examines the theoretical and quantitative implications of an intertemporal banking system for GHG emissions. In an article to be published in *Environmental and Resource Economics* (19(3):229-256), 2001, Leiby and Rubin show that environmental regulators can achieve the socially optimal level of emissions and output through time by setting the correct total sum of allowable emissions, and specifying the correct intertemporal trading ratio for banking and borrowing. For the case of a slowly decaying stock pollutant such as greenhouse gases, we show that the optimal growth rate of permit prices, and therefore the optimal intertemporal trading rate, has a closed-form solution equal to the ratio of current marginal stock damages to the discounted future value of marginal stock damages, less the decay rate of emissions in the atmosphere. In the practical situation where a time path of allowable emissions has already been negotiated, we show how an appropriately designed permit banking system has the potential to lower net social costs. Such a "second best" banking system is derived by adjusting the intertemporal trading ratio, taking into account the anticipated behavior of private agents. Simple numerical results are shown using a single-region simulation model to illustrate the potential gains from various possible banking systems. Ongoing work with the MERGE (Manne and Richels) and RICE (Nordhaus and Boyer) models seeks to extend these results to regional and intertemporal permit banking and borrowing.

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1.0 Introduction

Following the signing of the Framework Convention on Climate Change at the 1992 United Nations Conference for Environment and Development in Rio, which calls for the stabilization of greenhouse gas concentrations in the atmosphere at 1990 levels, a growing number of researchers and policy makers have proposed permit trading in greenhouse gasses (GHGs) (e.g., Falk and Mendelsohn (1993), Hahn and Stavins (1993), Swart (1993), Kosobud et al. (1994), Jackson (1995)). While appropriately recognizing the stock nature of the problem, none of this research has investigated the properties of intertemporal GHG permit trading in a general framework that allows the flexibility afforded when permits may be traded, banked and, possibly, borrowed.

At the same time, however, recent work has begun to investigate the properties of intertemporal permit systems for *flow* pollutants; pollutants whose deleterious effects are solely a function of the current flow rate (Rubin and King (1993), Biglaiser et al. (1995), Cronshaw and Kruse (1996), Rubin (1996), Kling and Rubin (1997)). In examining flow pollutants, these papers use intertemporal models which allow firms to bank (Cronshaw and Kruse (1996)) and bank and borrow (Rubin and Kling (1993), Rubin (1996)) emissions through time in addition to the inter-firm trading which characterizes single-period permit systems. Kling and Rubin (1997) show that unrestricted emission banking and borrowing of flow pollutants is not necessarily socially optimal. This arises because unrestricted permit banking and borrowing causes discounted permit prices and, therefore, discounted marginal abatement costs to be equalized through intertemporal arbitrage by private agents. At the same time, however, there is no reason to presume that the resulting emissions path is socially optimal, since the social optimum requires, for stationary damage and costs functions, that current value marginal abatement costs should be constant across time. However, as Kling and Rubin (1997) show, the banning of flow permit banking and borrowing is also not optimal.[1]

Most greenhouse gases are *stock*, rather than flow, pollutants. That is, the damage from their emission depends principally upon their accumulated stock in the environment. In the case of a stock pollutant permit banking is even more problematic. For stock pollutants, there is no reason to believe that marginal damages are equal in different periods. Indeed, the behavior of individual agents (firms or nations) can well diverge from the social optimum when intertemporally trading stock pollutants.

Permit systems that allow banking and borrowing (hereafter bankable permits) are seeing growing regulatory interest both nationally and internationally. The sulfur dioxide trading program, authorized by the 1990 Clean Air Act Amendments, is the best known and most

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[1]Biglaiser et al. also show that an intertemporal permit trading will not be optimal. The model used by Biglaiser et al., however, does not allow for the borrowing and banking of permits, but rather looks at trading lifetime rights to emit flow pollutants.

extensive venture in marketable permits to date.[2] This program allows firms to bank, but not borrow, permits. Another domestic example is current fuel economy regulations that allow automobile manufacturers to bank *and borrow* fuel economy credits for up to three years (49 USC 32903). Certainly, however, the grandest use yet envisioned for marketable permits is contained in the recent draft proposal by the U.S. Department of State to allow nations of the world to trade, bank and borrow greenhouse gas permits under the Framework Convention on Climate Change (USDOS, 1997).

Despite the reluctance of developing nations to allow any form of emission trading, the Kyoto Protocol signed last year does allow emission trading among Annex B (industrialized) nations (United Nation, Article 16bis 1997). The details on emission trading are to be negotiated in the future. Whether or not the banking and borrowing, in particular, of greenhouse gas permits will be allowed is not yet determined. A carefully structured trading system that makes some provision for permit banking and borrowing has significant merit. It can maintain market incentives for efficient emission reductions while still providing individual parties the time-flexibility that they may need to meet their negotiated obligations.

To date, international attention has rightly focused on the setting the initial endowments of permits and on the rules to insure that promised reductions do in fact occur. Overlooked, however, is what should happen if reduction goals are not met or are exceeded, and how one might set the rules in a situation where pressure for intertemporal permit banking and borrowing arises naturally. This research examines how environmental regulators can achieve the socially optimal level of emissions and output through time by setting the correct total sum of allowable emissions, and specifying the correct intertemporal trading ratio for banking and borrowing. For intertemporal greenhouse gas permit trading, we show that the optimal growth rate of permit prices, and therefore the optimal intertemporal trading rate, has the closed-form solution equal to the ratio of current marginal stock damages to the discounted future value of marginal stock damages less the decay rate of emissions in the atmosphere. Given the more realistic case of a non-optimal negotiated emission path, we then derive a permit banking system that has the potential to lower net social costs by adjusting the intertemporal trading ratio taking into account the behavior of private agents.

We report here on a series of numerical calculations of the gains from using bankable permit system given an non-optimal allocation of permits. We present the results from 2 models, a single-region model based on an early version of DICE (Nordhaus 1994, Nordhaus and Yang 1996), and a multi-region model MERGE (Manne and Richels 1997).

[2]See Burtraw (1994) and USGAO (1994) for overviews of the sulfur dioxide trading program.

2.0 Policy Setting and Theoretical Background for GHG Permit Trading

The theoretical literature has established a number of important properties regarding the efficiency and optimality of marketable emission permits.[3] One of the most important properties of marketable emission permits is that, for any given emission standard, a permit system can achieve that standard at least abatement cost (Montgomery, 1972). When pollution damages are due to the pooled effects of emissions from all firms, marketable permits can improve social welfare relative to a standard. They achieve the same emission level, and therefore the same damages, at lower cost. The efficiency gains arise because firms are allowed to move permits between sources so that firms equate marginal abatement control costs.

Sections 3 through 5 of this paper consolidate the intertemporal permit literature by developing and solving a generalized intertemporal permit system for emissions that both cause damage instantaneously (e.g., flow pollutants), and also accumulate in the environment such that damages also depend on the accumulated stock. Examples of this type of pollutant include the criteria pollutants carbon monoxide, nitrogen oxides, and non-methane volatile organic compounds that can cause acute health affects *and* can promote the atmospheric concentrations of greenhouse gases, including carbon dioxide, methane, and ozone (EIA, pp. xiv, 61, 1995).[4] This generalized permit system contains the results of Falk and Mendelsohn (1993), Rubin (1996), and Kling and Rubin (1997) as special cases.[5]

We turn in Section 6 to a permit system for the special case of emissions that only cause stock damages. This simpler case corresponds roughly to the greenhouse gas emission reduction regime proposed by the U.S. Department of State. For this simpler case we are able to derive a closed-form solution for the intertemporal permit trading ratio that induces decentralized agents to emit pollution at the socially optimal rate through time. For greater generality, but somewhat as an aside, in Section 7 we show the if private and social discount rates differ by some percentage, then the efficient banking rates should be increased by that percentage.

In addition, this paper examines intertemporal permit trading in the policy-relevant situation where a particular time path of permit allocations is already in place, perhaps due to political or diplomatic negotiations. In this situation, the time path is likely to differ from the social optimum. The questions we address are: how can banking systems be used to improve net social

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[3]See Cropper and Oates (1992) for a thorough review of the literature.

[4]The magnitude of the global warming potential of criteria pollutants, however, depends on local atmospheric conditions (EIA, p. 61, 1995).

[5]Although not shown, this same model structure, but using permanent as opposed to temporary permits, reproduces the results of Biglaiser et al. (1995). The results of Cronshaw and Kruse are also contained as a special case, but naturally have a different flavor since they arise from a discrete-time model.

welfare (Section 8); and, what are the magnitudes of potential gains (Section 9)?

The preliminary numerical estimate of potential gains from permit banking in Section 9 considers the particular case where greenhouse gas permits are allocated to stabilize emissions at the 1990 level for the next 150 years. This is a rough approximation of the recently negotiated Kyoto agreements, extended over a longer horizon, and applied to *all* regions. Given plausible parameter estimates taken from the literature, we compare the ability of various banking regimes with fixed and flexible trading rates to improve social welfare, for a range of sensitivity cases.

3.0 Intertemporal Emission Allocation From the Perspective of Society

In the social problem, the environmental regulator's objective is to maximize consumer and producer surplus less social damages from the good, $y(t) = \sum_{i=1}^{N} y_i(t)$, whose production causes instantaneous emission flows, $E(t) = \sum_{i=1}^{N} E_i(t)$, and cumulative emission stock *S(t)*.[6] Unlike flow pollutants, stock pollutants accumulate in the environment because their rates of emission into the environment exceed the environment's assimilative capacity.

Letting $S(t)$ be the total stock of all firms' emissions at any point in time, $S(t) = \sum_{i=1}^{N} S_i(t)$, we see below in (1) that whenever the sum of all firms' emission is greater than the natural decay of emissions, $\gamma S(t)$, then the stock of emission will be increasing. Here, emissions are taken to decay at a constant rate γ.[7]

$$\dot{S}(t) = \sum_{i=1}^{N} E_i(t) - \gamma S(t) \tag{1}$$

With flow pollutants, damages are solely a function of their instantaneous emission flows (rates). More generally, damages from stock pollutants are a function, at each instant in time, of the level of accumulated pollution and possibly the contemporaneous flow.

Emissions are assumed to harm social welfare as described by the convex damage function $D(E(t), S(t),t)$, where $D_E(E,S,t) > 0$, $D_{EE}(E,S,t) > 0$, $D_S(E,S,t) > 0$, $D_{SS}(E,S,t) > 0$, and $D_{ES}(E,S,t) > 0$. With a stock pollutant, damages do not stop at the end of the end of the regulatory program as denoted by the end of the time horizon, T. Damages from the stock of pollution will continue until the stock pollutant decays to benign levels. The final-value term $F(S(T))$ captures the value of damages for all time periods after T (measured in period T dollars).

Firms are assumed to produce a good, y_i, and emissions, E_i. There are N heterogeneous firms

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[6] Symbols subscripted by ***i*** indicate variables for individual firms, otherwise the symbols refer to market totals.

[7] In general, of course, the rate of decay need not be constant. This simplification does not substantially affect our analysis.

who are price takers in their input and output markets.[8] Firm *i*'s minimum total cost of producing output $y_i(t)$ and emission level $E_i(t)$ is $C_i(y_i(t),E_i(t),t)$. It is assumed that $C_i(y_i(t),E_i(t),t)$ is strongly convex in $(y(t),E(t))$ and with $C_y > 0$, $C_{yy} > 0$, $C_E < 0$, $C_{EE} > 0$ and $C_{yE} < 0$.[9] Therefore, higher levels of emissions are associated with lower production costs both total and at the margin. Given this notation, marginal abatement costs are denoted as $-C_E > 0$.

In sum, the environmental regulator's job is to balance output, control costs and pollution damages through time. From a multi-year (continuous time, finite horizon) perspective, the welfare maximization problem is given below. In addition to the terms already defined, *B(y)* is the benefit of consumption of good y, and $e^{-\rho t}$ is the instantaneous social (or collective) discount factor.[10]

$$J^* \equiv \max_{\substack{E_1 \ldots E_N \\ y_1 \ldots y_N}} \int_0^T e^{-\rho t}\left(B(y(t)) - \sum_{i=1}^{N} C_i(y_i,E_i,t) - D(E,S,t) \right) dt - e^{-\rho T}F(S(T))$$

$$s.t.: \qquad \dot{S}(t) = E(t) - \gamma S(t) \qquad (2)$$

$$y_i(t) \geq 0, \quad E_i(t) \geq 0$$

The current value Hamiltonian for this problem is (after substituting for *e*, *S* directly and assuming that the non-negativity bounds on emissions and output do not bind):

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[8]In the context of global warming each "firm's" emissions and output can be interpreted as each nation's emissions and output. The coordinating authority is not a single nation's government, but the UN member states acting collectively.

[9]Here, subscripts which are variable names refer to the partial derivative with respect to that variable. In addition, the "i" subscripts indicating the firm under consideration and the functional dependency of variables on t will frequently be suppressed to reduce clutter.

[10]Since, for the GHG control problem, agents are countries with different preferences, the construction of a single global objective is problematic. Alternatively, one could explore several Pareto-efficient outcomes for 'collectively efficient' emissions. However, for this paper we follow the common practice in environmental economics of first exploring and contrasting the fully cooperative case with the independent action case. This global optimization approach is also consistent with many of the compact climate-economic models that might be used for numerical evaluation.

$$H = B(y(t)) - \sum_{i=1}^{N} C_i(y_i, E_i, t) - D(\sum_{i=1}^{N} E_i, \sum_{i=1}^{N} S_i) + \Lambda\left(\sum_{i=1}^{N} E_i - \gamma S\right) \qquad (3)$$

The first order necessary conditions for all *i* and *t*, are:

$$\frac{\partial H}{\partial y} = \frac{\partial B}{\partial y} = P_y(y) - \frac{\partial C_i}{\partial y_i} = 0 \qquad (a)$$

$$\frac{\partial H}{\partial E_i} = -\left(\frac{\partial C_i}{\partial E_i} + \frac{\partial D}{\partial E_i}\right) + \Lambda = 0 \qquad (b)$$

$$-\frac{\partial H}{\partial S} = \dot{\Lambda} - \rho\Lambda = \frac{\partial D}{\partial S} + \Lambda\gamma \qquad (c) \qquad (4)$$

$$\Lambda(T) = -\frac{\partial F(S(T))}{\partial S(T)} \qquad (d)$$

Note: In the body of the text, we denote the shadow cost of a unit of stock ($-\Lambda(t)$) by $f_S(t)$, to indicate that it reflects the marginal stream of future damages.

Since damages depend on the total current emissions *e* and stock *S* for all firms, marginal flow and stock damages are equal for all *i*.

Solving the differential equation (4) (c) with terminal condition (4)(d) yields the current shadow cost of a unit of pollution stock, which we denote at time *t* as $f_S(t)$.

$$f_s(t) = \int_t^T e^{-(\rho+\gamma)(\tau-t)} \frac{\partial D}{\partial S} d\tau + e^{-(\rho+\gamma)(T-t)} \frac{\partial F(T)}{\partial S} \qquad (5)$$

Since our objective is to maximize output value net of costs, the current shadow value of a unit of pollutant stock is negative. In fact, the current marginal value of a unit of pollutant stock is equal to minus the net present value of the future stream of induced marginal damages, up to and including the marginal final damages $\partial F(T)/\partial S$. This stream of marginal damages is discounted at the social rate ρ and decayed at rate γ, for a combined discount rate of $\rho+\gamma$. At the terminal time $f_S(T) = \partial F(T)/\partial S$.

Note that $f_S(t)$ depends on the future time path of the stock, and we are interested in its value along the socially optimal path.[11] Along the socially optimal path, then, the following conditions (derived from the first order conditions) must be met:

[11]The single asterisks indicate that the variables are being evaluated at their optimal levels for the social, or cooperative, problem.

$$
\begin{aligned}
&\frac{\partial C_i^*}{\partial y_i} = \frac{\partial B^*}{\partial y_i} = P_y(y) && (a)\\
&-\frac{\partial C_i^*}{\partial E_i} = \frac{\partial D^*}{\partial E} + f_S(t) && (b)\\
&\dot{f}_S = (\rho+\gamma)f_S - \frac{\partial D^*}{\partial S} \quad \text{and} \quad f_S(T) = \frac{\partial F(T)^*}{\partial S(T)} && (c)\\
&\text{i.e.} \quad f_S = \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^*}{\partial S}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F(T)^*}{\partial S} && (d)
\end{aligned}
\qquad (6)
$$

As expected, first order condition (a) requires that each firm's marginal production costs equal the marginal value of output in every period. Firms will assure this on their own, provided that the price of the output reflects its social value. First order condition (b) says that the regulator should choose an emission path $E_i^*(t)$ for firm i such that at each instant over the planning horizon marginal abatement costs equal the marginal (flow) damages plus the current shadow cost of another unit of pollutant stock. Mathematically, one sees from (d) that the current shadow cost must be positive if marginal stock damages are positive. This means that in the presence of stock damages, marginal control costs should be at least as great as marginal flow damages from the current flow of emissions. From (c) we see that the shadow cost of a unit of the stock pollutant may rise or fall through time.

Differentiating (b) with respect to time and substituting in (c) yields a recursive or differential statement of the optimal emissions control path

$$
\frac{\partial[C_i(y_i^*,E_i^*,t)+D(S^*,E^*,t)]}{\partial E} = \frac{1}{(\rho+\gamma)}\left[-\frac{\partial D(S^*,E^*,t)}{\partial S} + \frac{d}{dt}\left(\frac{\partial[C_i(y^*,E_i^*,t)+D(S^*,E^*,t)]}{\partial E}\right)\right]. \qquad (7)
$$

This form shows how current net marginal costs and damages trade off with the present value of future costs and damages. The present value calculation is based on an infinitely lived annuity that declines at the decay rate γ and is discounted at rate ρ. An interesting alternative form shows how abatement costs are balanced against damages. Marginal abatement costs minus the present value of changes in marginal abatement costs through time should be equal to marginal damages from emissions plus the present value of marginal damages from an increase in the stock of pollution.

$$
-\frac{\partial C_i^*}{\partial E_i} - \frac{1}{(\rho+\gamma)}\frac{d}{dt}\left(\frac{-\partial C_i^*}{\partial E_i}\right) = \frac{\partial D^*}{\partial E} + \frac{1}{(\rho+\gamma)}\left(\frac{\partial D^*}{\partial S} - \frac{d}{dt}\left(\frac{\partial D^*}{\partial E}\right)\right) \qquad (8)
$$

This result is an extension of the result in Falk and Mendelsohn (1993:78), to the case where damages may depend on both emissions flows and stocks and the terminal value of emissions is considered.

4.0 Regulation of Emission Flow and Stocks Via Permits

In principle, an agency (national or international) regulating a stock pollutant might only be concerned that the integral of emissions -- less stock decay -- be less than a given standard $\bar{S}(t)$ at each point in time t (e.g. equation (9)).[12]

$$S(t) = \int_0^t \left(E(\tau) - \gamma S(\tau)\right) d\tau + S(0) \leq \bar{S}(t) \quad \forall\ t \tag{9}$$

Alternatively, as suggested by Kosobud et al. (1994), a regulatory agency could set a series of maximum emission rates, $\bar{E}(t)$, requiring $E(t) \leq \bar{E}(t)$ at each time t. If the emission limits were constant through time this would assure (in the context of global warming) that developed nations would freeze their rates of emission of greenhouse gases.

We address this potential ambiguity in regulatory design by supposing that the environmental authority has a standard for both the instantaneous emission flow and the cumulative stock of pollution in the environment at any point in time. We consider both standards which are derived from the social optimal control problem (Eq. 2) and standards which are "negotiated" or possibly arbitrary. Since there need not be a unique relationship between a stock standard and a flow standard, an environmental authority could establish separate, independent standards. By independent, we simply mean that at any point in time one standard may bind while the other may not. Obviously though, the two standards are not wholly unrelated to each other. For example, a very strict flow standard could insure that the cumulative stock standard is never a binding constraint, while a relatively loose flow standard would not. Regardless of the type of emission standard, be it based on a stock or a flow, the regulatory agency may allow firms to bank and borrow permits. Indeed it turns out that banking and borrowing, subject to intertemporal trading rates , are necessary for a decentralized permit system to attain the socially optimal emission path through time.

Flow Permit Bank

Denoting the flow emission permits a firm purchases by $\boldsymbol{x}(t)$, we can define the flow permit bank $B_{E_i}(t)$, as the cumulative difference between a firm's endowed emission flow limits, $\bar{E}_i(t)$, and its actual emission flow level, $E_i(t)$, plus any purchased flow permits, $x_i(t)$:

$$\begin{aligned} B_{E_i}(t) &= \int_0^t (\bar{E}_i(\tau) - E_i(\tau) + x_i(\tau)) d\tau + B_{E_i}(0) \\ \dot{B}_{E_i}(t) &= \bar{E}_i(t) - E_i(t) + x_i(t) \end{aligned} \tag{10}$$

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[12]Kosobud et alias (1994) use a terminal stock standard to determine the total number of permits to allocate to firms. The precise form of their standard is not specified.

Stock Permit Bank

Within this context, the stock pollution bank, $B_{S_i}(t)$, can also be is defined as the time-integral of the difference between the firm's endowed stock standard, $\overline{S}_i(t)$, and its actual stock level, $S_i(t)$, plus any stock permits, $z_i(t)$, purchased or sold (11). In the most general case, firms may not be required to have sufficient permits at any point in time to cover their contemporaneous stocks, that is they may borrow or bank permits that must later be repaid.

$$B_{S_i}(t) = \int_0^t \left(\overline{S}_i(t) - S_i(t) + z_i(\tau)\right) d\tau + B_{S_i}(0)$$
$$\dot{B}_{s_i}(t) = \overline{S}_i(t) - S_i(t) + z_i(t) \tag{11}$$

For concreteness, emission flows, E_i, could be measured in tons per year; stocks, S_i, would thereby be measured in tons. Accordingly, the flow bank account would then be measured in tons, and the stock permit bank account in ton-years.

For the flow and stock permit banks defined here, the intertemporal trading ratio (ITR) for permits is unitary: permits may be borrowed and saved freely on a one-to-one basis. Another way of saying this is that there is a zero banking "interest" rate offered for saving or charged for borrowing. We will turn to a determination of the socially optimal banking trading rates in Section 4.

Flow and Stock Permit Banking and Borrowing With 1-to-1 Intertemporal Trading Ratio

Shown below in (12) is the firm's problem of maximizing profits subject to emission and stock constraints. At every point in time each firm is allocated emissions flow permits $\overline{E}_i$ and stock permits $\overline{S}_i$. These permits may be banked or borrowed at a 1-to-1 exchange ratio, subject to the bank equations of motions and non-negativity constraints. Firms may also purchase or sell permits for either stock or flow pollution at the (market determined) prices P_s and P_E respectively. The bank balances must be nonnegative at the terminal time T.

$$J_i^{**} = \underset{y_i, E_i, x_i, z_i}{Max} \int_0^T e^{-\rho t}\left\{P y_i - C_i(y_i, E_i, t) - P_E x_i - P_s z_i\right\} dt$$

$$
\begin{aligned}
s.t.: \quad & \dot{S}_i = E_i - \gamma S_i \\
& \dot{B}_{s_i} = \overline{S}_i - S_i + z_i \\
& \dot{B}_{E_i} = E_i - E_i + x_i \\
& x_{\min_i} \le x_i \le x_{\max_i} \\
& z_{\min_i} \le z_i \le z_{\max_i} \\
& y_i \ge 0, \quad E_i \ge 0, \quad B_{E_i}(T) \ge 0, \quad B_{S_i}(T) \ge 0
\end{aligned}
\tag{12}
$$

The current value Hamiltonian (ignoring non-negativity multipliers on output, emissions and permit bounds) is:

$$H = P_y y_i - C(y_i,E_i,t) - P_E x_i - P_s z_i + \lambda_{S_i}(E_i - \gamma S_i) + \lambda_{E_i}(\bar{E}_i - E_i + x_i) + \lambda_{k_i}(\bar{S}_i - S_i + z_i) \qquad (13)$$

The costate variable on the emission stock transition equation is λ_s. The costate variable for the emission flow and stock bank accounts are λ_{ei} and λ_{ki}, respectively. The constrained profit maximizing behavior still adjusts output until marginal cost equals product price (as long as output is nonzero). The necessary conditions are

$$\begin{aligned}
\frac{\partial H}{\partial y_i} &= P_y - \frac{\partial C}{\partial y_i} = 0 \\
\frac{\partial H}{\partial E_i} &= \lambda_{s_i} - \lambda_{E_i} - \frac{\partial C}{\partial E_i} = 0 \\
\frac{\partial H}{\partial x_i} &= -P_E + \lambda_{E_i} = 0 \\
\frac{\partial H}{\partial z_i} &= -P_s + \lambda_{K_i} = 0 \\
-\frac{\partial H}{\partial S} &= \dot{\lambda}_S - \rho\lambda_{S_i} = \gamma\lambda_{S_i} + \lambda_{K_i} \\
-\frac{\partial H}{\partial B_{E_i}} &= \dot{\lambda}_{E_i} - \rho\lambda_{E_i} = 0 \\
-\frac{\partial H}{\partial B_{S_i}} &= \dot{\lambda}_{K_i} - \rho\lambda_{K_i} = 0
\end{aligned} \qquad (14)$$

Solving the first order necessary conditions and rearranging as above yields the following expression:[13]

$$-\frac{\partial C_i^{**}}{\partial E_i} - \frac{1}{(\rho+\gamma)}\frac{d}{dt}\left(\frac{-\partial C_i^{**}}{\partial E_i}\right) = P_E^{**} + \frac{1}{\rho + \gamma}\left(P_s^{**} - \frac{d}{dt}P_E^{**}\right). \qquad (15)$$

It is optimal for the firm to expand emissions until the current marginal abatement costs minus the present value of changes in marginal abatement costs are equal to the price of a flow pollution permit plus the present value of a stock pollution permit and the present value in future changes in the price of flow pollution permits. Here the present value calculation is based on an infinitely lived annuity that declines at the decay rate γ and is discounted at rate ρ.

The private optimality condition has the integral solution, which, for a permit program of finite

[13]The double asterisks indicates that all the variables are evaluated at their privately optimal levels. We do not yet consider the possible divergence between the social and private rate of discount, but do so later in Section 6.

duration T, is:

$$C_{ia}^{**}(t) \equiv -\frac{\partial C_i^{**}}{\partial E_i} = P_E + \int_t^T e^{-(\rho+\gamma)(\tau-t)} P_S(\tau) d\tau \tag{16}$$

This reveals that private marginal abatement costs should equal current flow permit price plus the NPV cost of stock permits needed to cover the resultant increment of pollutant stock as it decays over time at rate γ.

Since there is trading in each period, all firms face the same permit prices. Given intertemporal trading, however, the permit prices are not independent across time. When firms have non-bounded solutions, then the following market outcomes for permit price paths are necessary for market equilibrium and can be derived from differentiating and manipulating the first order conditions.

$$\dot{P}_E^{**} = \rho P_E^{**}, \qquad \dot{P}_S^{**} = \rho P_S^{**} \tag{17}$$

We now see that when firms are allowed to freely borrow and bank stock and flow permits through time, with a unitary ITR, market permit prices (and marginal control costs) will rise at the rate of discount. This is essentially a permit arbitrage condition. Substituting $\dot{P}_E^{**} = \rho P_E^{**}$ into (15) we derive the following result.

$$-\frac{\partial C^{**}}{\partial E_i} - \frac{1}{(\rho+\gamma)}\frac{d}{dt}\left(\frac{-\partial C^{**}}{\partial E_i}\right) = \frac{1}{\rho+\gamma}(\gamma P_E^{**} + P_S^{**}) \tag{18}$$

With flow and stock permits, the firm equates its marginal abatement costs less the present value of changes in marginal abatement costs to a weighted combination of flow and stock permit prices. The combination reflects current and anticipated flow permit prices and the present value of induced future stock permit costs. This result generalizes those of Rubin (1996) and Kling and Rubin (1997) by including stock damages.

5.0 Optimal Intertemporal Trading Rates with Stock and Flow Permits

For the case of stock and flow permits, matching private abatement efforts (15) to the socially optimal abatement efforts (8) requires that permit prices satisfy:

$$P_E^{**} + \frac{1}{\rho+\gamma}\left(P_S^{**} - \frac{d}{dt}P_E^{**}\right) = \frac{\partial D^*}{\partial E} + \frac{1}{\rho+\gamma}\left(\frac{\partial D^*}{\partial S} - \frac{d}{dt}\frac{\partial D^*}{\partial E}\right). \tag{19}$$

The flow pollution permit price plus the present value of the stock pollution permit price equals the weighted sum of stock and flow marginal damages. The obvious solution would be to set each permit price equal to the marginal damage for the respective variable (flow or stock):

$$P_E^{**} = \frac{\partial D^*}{\partial E}, \qquad P_S^{**} = \frac{\partial D^*}{\partial S} \tag{20}$$

Unfortunately this effort will be frustrated by the market arbitrage outcome that requires permit prices to rise at the discount rate, see (17). Alternatively, if banking and borrowing are prohibited, then permit prices will fluctuate through time depending on the permit endowment at every point in time. These permit price fluctuations will not, unless by accident, yield the correct intertemporal paths for emissions and stocks.

Consider now, that rather than allowing permits to trade on a one-to-one basis through time, that some trading rate is applied whereby permits do not have the same value when used or saved at different points in time. Altering the trading rate is equivalent to altering the rate of change in discounted permit prices for different points in time, and can, in principle, direct firms to borrow and bank at a socially optimal rate. Of course the correct amount of permits must also be issued to get the *level* of permit prices correct. Since the number of permits allocated in each period is often a negotiated outcome, those permit allocations may diverge substantially from the social optimum. By introducing a banking regime and altering the trading ratio, the regulatory authority can help correct for non-optimal permit endowments over time.

The simplest way to adjust ITRs is to include "interest" on permit bank account balances. Since bank account balances can be positive (saving) or negative (borrowing), a positive interest rate would reward saving and discourage borrowing. It would also imply that one permit saved now could be exchanged for more than one permit later. It is simple to include the banking rates or "interest" payments in the flow and stock permit bank account dynamic equations:[14]

$$\begin{aligned} \dot{B}_{E_i} &= \bar{E}_i - E_i + x_i + r_E B_{E_i} \\ \dot{B}_{S_i} &= \bar{S}_i - S_i + z_i + r_S B_{S_i} \end{aligned} \tag{21}$$

The current-value Hamiltonian for the flow and stock permit case, including intertemporal banking charge rates r_e and r_s is:

$$\begin{aligned} H_i = {} & P_y y_i - C_i(y_i, E_i, t) - P_{Ex_i} - P_s z_i) + \lambda_S(E_i - \gamma S_i) \\ & + \lambda_{E_i}(\bar{E}_i - E_i + x_i + r_E B_E) \\ & + \lambda_K(\bar{S}_i - S_i + z_i + r_s B_{S_i}) \end{aligned}$$

[14]Kling and Rubin (1997) suggest an alternative way to represent the uneven trading rate between flow permits: $\dot{B}_{Ei}(t) = (E_{bi}(t) - E_i(t) + x_i(t))\,\rho(t)$. If we restrict $r_E = \rho$ (as do Kling and Rubin) then both flow banking equations produce essentially the same results with some accounting differences. Our suggested method significantly generalizes their results (shown below) by deriving optimal ITRs where r_E^* and r_S^* generally do not equal ρ. In addition, our suggested method also more explicitly emphasizes that the interest payment or charge accrues to the store of banked or borrowed emissions or stocks.

These alterations of the bank accounts equations of motion lead to the same optimality conditions as the even-exchange trading and banking case, except for those conditions related to the time-rate of change of the shadow price of bank accounts. The new equations of motion for the bank accounts' state and costate variables are:

$$
\begin{aligned}
-\frac{\partial H}{\partial B_E} &= \dot{\lambda}_E - \rho\lambda_E = -r_e\lambda_e \\
-\frac{\partial H}{\partial B_S} &= \dot{\lambda}_K - \rho\lambda_K = -r_S\lambda_K
\end{aligned}
\tag{23}
$$

The new percentage rates of change of the permit prices (indicated by a "hat" (^)) symbol are now:

$$
\begin{aligned}
\frac{\dot{\lambda}_E}{\lambda_E} &= \frac{\dot{P}_E}{P_E} \equiv \hat{P}_E = \rho - r_E \\
\frac{\dot{\lambda}_K}{\lambda_K} &= \frac{\dot{P}_S}{P_S} \equiv \hat{P}_s = \rho - r_S
\end{aligned}
\tag{24}
$$

Here we see that for firms to have a non-bounded, internal solution permit prices must grow at the rate of discount *less* the rate of interest charged or paid on borrowed or banked emissions. Thus, the effect of a positive interest rate is to offset the discount rate and reduce the growth rate of market permit prices.[15] This means that present value marginal abatement costs will decline through time relative to the zero-interest case. The only way for this to happen, ceteris paribus, is for emissions to increase through time faster than they would have with one-to-one intertemporal permit trading. Thus, an effect of paying positive "interest" on bank holding is, as one would suspect, to encourage extra emission reductions early in the T period time horizon.

Social optimality can be achieved under this system if, at every point in time, private emissions, E_i^{**}, (or marginal abatement costs) are identically equal to the socially optimal emissions, E_i^{*}, (abatement costs) for every firm. This is true when the left-hand-sides of (15) and (8) are equal. Accordingly, their right-hand-sides should be equal as well. Thus, optimal permit prices should equal optimal marginal damages and, by choosing the trading ratio correctly, the percentage change in permit prices should equal the percentage change in marginal damages.

---

[15]The slower growth rate of permit prices is achieved when positive banking interest rates promote early permit saving, effectively increasing the future supply of permits.

$$
\begin{aligned}
P_E^* &= \frac{\partial D^*}{\partial E} \\
P_S^* &= \frac{\partial D^*}{\partial S} \\
r_E^* &= \rho - \hat{P}_E^* = -\frac{dln}{dt}(e^{-\rho t}P_E^*) = -\frac{dln}{dt}\left(e^{-\rho t}\frac{\partial D^*}{\partial E} \right) \\
r_S^* &= \rho - \hat{P}_S^* = -\frac{dln}{dt}(e^{-\rho t}P_S^*) = -\frac{dln}{dt}\left(e^{-\rho t}\frac{\partial D^*}{\partial S} \right)
\end{aligned}
\tag{25}
$$

This says that the optimal interest rates on the dual flow and stock bank accounts are opposite in sign and equal in magnitude to the time rate of change of the discounted marginal flow and stock damages, respectively. The interest rate of bank accounts ought to be positive when the time rate of change of discounted damages is negative (i.e., when the present value of marginal damages declines through time). Note that if the socially optimal marginal damages are constant in discounted terms, then r_E and r_s should be identically zero, i.e., no interest should be yielded on bank accounts. If discounted marginal damages rise through time on the socially optimal path, then the optimal interest rate on stock bank accounts would be negative. For example if population increases cause optimal marginal damages to rise quickly, a negative interest rate would be desired to encourage firms to borrow, emitting more now in exchange for paying back (emitting less) later.

An important consideration in implementing such a system is the amount of information that must be known by the environmental regulator. In order to set the permit prices and time rates of change in permit prices optimally, the environmental regulator must know the marginal damages and time rates of change in marginal damages from flows and stocks evaluated at their optimal level. This requires knowing aggregate marginal abatement costs. Of course if the environmental regulator knew each firm's marginal abatement costs (as well as optimal marginal damages) then consistent with static analysis, permits, taxes and standards are all equivalent. Moreover (and again consistent with static analysis) if the environmental regulator is able to estimate aggregate marginal abatement cost and damage functions, then this optimal permit system can be implemented without the detailed standard setting required of command-and-control systems. On the other hand, the regulator may prefer to simply issue the optimal number of permits in each period and not allow banking and borrowing.

The case for intertemporal permit systems that allow banking and borrowing is much stronger when considering non-optimal permit endowments as in the case of the global warming treaty arising from the Kyoto convention. After describing how to set up a permit system for GHG emissions that attains a social optimum (Sections 5 and 6), we take up the potentially more policy relevant issue of how to use a permit system for GHG emissions starting from a non-optimal endowment (Sections 7 and 8).

6.0 Optimal Intertemporal Permit Trading Rates for Greenhouse Gas Emission Flows

Consider now the different case where the pollutant causes only stock-related damages, and where the regulatory authority chooses to issue only flow permits. Banking in this case corresponds well to the (draft) greenhouse gases emissions trading system previously proposed by the U.S. Department of State as a means of fulfilling the U.S. commitment to the Framework Convention on Climate Change (USDOS, 1997). The question we would like to answer is, if flow permit banking and borrowing is allowed, how should the interest rate on bank accounts be set? For a stock-only pollutant, with no flow damages ($D_e = 0$), the socially optimal condition is:

$$-\frac{\partial C_i^*}{\partial E_i} - \frac{1}{(\rho+\gamma)}\frac{d}{dt}\left(\frac{-\partial C_i}{\partial E_i^*}\right) = \frac{1}{(\rho+\gamma)}\frac{\partial D^*}{\partial S}. \tag{26}$$

This implies a control path solution of the form:

$$-\frac{\partial C^*}{\partial E_i} = \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^*(\tau)}{\partial S}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F(S(T))}{\partial S}. \tag{27}$$

In words, this says that at any time in the planning horizon, the socially optimal emission level is chosen such that discounted marginal abatement costs for each firm equals the present discounted value of all future marginal stock damages over the planning horizon plus the present value of marginal terminal stock damages that occur beyond the regulatory time horizon. Note that the "discount" rate used is ($\rho+\gamma$), the financial discount rate plus the stock decay rate.

In the case of flow-only permits, where the stock permits are essentially unlimited, or $P_S = 0$, the market trading outcome would yield private abatement to the extent for each firm and time marginal abatement costs equal the current market price of a flow permit: $-C_{E_i}^{**} = P_E^{**}$. This repeats the usual static result. The regulatory authority, therefore, can induce firms to control their emission in a socially optimal manner if it can assure that the market permit price matches the socially optimal price, i.e. $P_E^{**} = P_E^* = C_E^*$, where the socially optimal price P_E^* reflects the present value of future marginal stock damages:

$$P_E^* = f_s \equiv \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D(\tau)^*}{\partial S}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F(S(T)^*)}{\partial S} \tag{29}$$

Taking the time derivative, the optimal permit price path ought to be:

$$\begin{aligned}\dot{P}_E^*(t) &= -e^{-(\rho+\gamma)(t-t)}\frac{\partial D(t)}{\partial S} + \int_t^T(\rho+\gamma)e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D(\tau)}{\partial S}d\tau + (\rho+\gamma)e^{-(\rho+\gamma)(T-t)}\frac{\partial F(S(T))}{\partial S} \\ &= -\frac{\partial D(t)}{\partial S} + (\rho+\gamma)P_E(t).\end{aligned} \tag{30}$$

The socially optimal time path of flow permit prices, therefore, depends on the discount rate, the stock decay rates, and the marginal stock damage at every point in time. Moreover, the proportional time rate of change in socially optimal permit prices is given by:

$$\frac{d\ln P_E^*}{dt} \equiv \hat{P}_E^* = (\rho+\gamma) - \frac{1}{P_E^*}\frac{\partial D^*}{\partial S}. \tag{31}$$

This result for permit price growth rates differs from those above because now we are using bankable *flow* permits alone to control a *stock* pollutant's effects. As before, the flow permit market outcome, with banking at interest rate r_E is:

$$\hat{P}_E^{**} = \rho - r_E \tag{32}$$

The regulatory authority, in seeking social optimality, should set the banking interest rate to assure coincidence of the market and socially optimal permit price paths, assuming that the starting permit price, as determined by the integral over time of all permit allocations, is optimal. This means that

$$\rho - r_E^* = \rho+\gamma - \frac{1}{P_E^*}\frac{\partial D^*}{\partial S}. \tag{33}$$

Substituting in for P_E^* from (29) the optimal banking rate, r_E*, for flow permits used to control damages from a stock pollutant is given by:

$$r_E^* = \frac{\frac{\partial D^*}{\partial S}}{f_S} - \gamma$$

$$\text{Where } f_S(t) \equiv \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^*(\tau)}{\partial S(\tau)}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F^*(S(T))}{\partial S(T)} \tag{34}$$

Interestingly, we see that the optimal banking rate equals the ratio of *current* marginal stock damages to the *discounted future value* of marginal stock damages less the decay rate of emissions in the atmosphere. Each of these factors varies with the level of stock emissions. These factors may also vary with technical advances in damage mitigation, change in population, and changes in ecosystem resiliency due to other stresses.

7.0 Permit Banking Rates When Private and Social Discount Rates Differ

The preceding results have not distinguished between the discount rates that may be used by individual agents (be they firms or governments) and the collective social planner (environmental regulator). For the purposes of planning a banking system, the key point is that it is the *private* discount rate that will determine the time path of permit prices (through private arbitrage in permit markets), while it is the *social* discount rate that should be used in determining optimal abatement costs and marginal damages. Suppose that the private discount rate, *i*, exceeds the social rate ρ. With unrestricted banking, in the market equilibrium, permit prices will grow at the private discount rate minus the flow permit banking interest rate r_e:

$$\hat{P}_E^{**} = i - r_E \tag{35}$$

In this case, the optimal banking interest rate is given by the condition:

$$i - r_E^* = -\frac{\frac{\partial D^*}{\partial S}}{f_S^*} + (\rho + \gamma). \tag{36}$$

Thus when the private and social (individual agent and collective group) discount rates diverge, the flow permit banking interest rate must be increased by their difference, i-ρ:

$$r_E^* = \frac{\frac{\partial D^*}{\partial S}}{f_S^*} - \gamma + (i - \rho) \tag{37}$$

8.0 Permit Banking with Non-Optimal Emission Limits

So far we have discussed how to achieve a socially optimal time path of emissions via banking and borrowing (intertemporal trading) of permits. We found that when banking and borrowing are allowed without restrictions, the total number of permits issued determines the permit price level and the banking interest rate, or ITR, determines the time-rate of change of permit prices. Together, these can be set to achieve a socially more efficient path. Here we take up the regulation of GHG emission via bankable permits when the total emission levels are set non-optimally via negotiation, as may be the case under the Kyoto Protocol.[16]

$$E_i(t) \le \bar{E}_i(t) \quad \forall i,t \tag{38}$$

Suppose, through negotiation, annual emission limits $\bar{E}_i(t)$ are set for each region i and time t. Based on individual incentives, each agent or region would choose to emit as much as possible, provided marginal abatement costs at the emission limit exceed the marginal damages *to that agent or region i alone*. If trading is allowed in each period, but banking and borrowing are not, in each period abatement costs between trading partners equalize and all permits are used.

Recall that along the socially optimal path, $E^*(t)$, $S^*(t)$, marginal abatement costs should equal current marginal flow damages plus the discounted sum of future marginal stock damages attributable to current emissions:

$$C_a^*(t) = D_E^*(t) + f_S(t) \tag{39}$$

Here the discounted future marginal stock damages $f_S(t)$ are evaluated along the future socially-optimal stock path, $S^*(t)$. Along the negotiated emissions path $\bar{E}(t)$, however, this efficiency condition will not be met. Rather, marginal abatement costs diverge from current plus future

[16]In principle, of course, the emission limits could be set optimally by negotiation, but this is unlikely especially in a intertemporal context.

$$\left(D_E(\underline{E}) + f_S(\underline{S})\right) - C_a(\underline{E}) \equiv \zeta(t) \begin{smallmatrix}\leq \\ \geq\end{smallmatrix} 0 \tag{40}$$

marginal damages by an amount ζ, with an indeterminate sign:

Allowing banking and borrowing of permits given a non-optimal standard has the potential to significantly reduce the net social cost of GHG abatement (private abatement costs plus remaining social damages) since the negotiated permit limits need not be optimal in terms of the cumulative sum of emissions and in terms of the time-placement of emissions. As we show below in our numerical simulations, even simple unitary (1:1) banking and borrowing can often have a lower social cost than no banking and borrowing at all.

As a benchmark thought experiment, it is worth defining the socially "second-best" reallocation of emission abatement across time. We define this second-best emission path to be one that minimizes discounted net social costs (abatement plus damage) while preserving the same cumulative emissions over time as those allowed under the negotiated emissions permits. This second-best emission path is the one that an environmental regulator would choose given that the regulator were able to choose *when* to allocate emissions and damages across time optimally, while preserving the cumulative emission target.

The "second-best" constrained optimization problem, reallocating a fixed total number of emission permits, is the same as the original social optimization problem in Equation 2 above, with the additional constraint on cumulative emissions over the time horizon: $\int_0^T E(t)dt \leq \int_0^T \bar{E}(t)dt$. This integral constraint on the control variable is best included in the optimization problem by the introduction of a new state variable, $E_c(t)$, which represents cumulative emissions up to time t, and a terminal constraint on $E_c(T)$:

$$\begin{aligned} E_c(t) &\equiv \int_0^t E(\tau)d\tau \quad \Leftrightarrow \quad \dot{E_c}(t) = E(t), \quad E_c(0) = 0 \\ E_c(T) &= \bar{E}_c \equiv \int_0^T \bar{E}(t)dt \end{aligned} \tag{41}$$

Adding these restrictions results in a new shadow cost, "θ" in the optimality condition for marginal abatement costs given in (42)b:

$$\frac{\partial C_i^+}{\partial y_i} = P_y(y) \qquad (a)$$

$$-\frac{\partial C_i^+}{\partial E_i} = \frac{\partial D^+}{\partial E} + f_S^+(t) - \theta(t) \qquad (b)$$

$$\dot{f}_S^+ = (\rho+\gamma)f_S^+ - \frac{\partial D^+}{\partial S} \quad \text{and} \quad f_S^+(T) = \frac{\partial F^+(T)}{\partial S(T)} \qquad (c) \qquad (42)$$

$$\text{i.e.} \quad f_S^+ = \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^+}{\partial S}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F^+(T)}{\partial S} \qquad (d)$$

$$\dot{\lambda}_S^+ = \rho\lambda^+ \Leftrightarrow \lambda^+(t) = e^{\rho t}\lambda^+(0) = e^{-\rho(T-t)}\lambda^+(T) \qquad (e)$$

Thus we see that in the second-best social optimum, given a constraint on the total emissions over the time horizon, the marginal abatement cost will no longer be set equal to the marginal flow damages plus marginal future stock damages. Rather abatement costs and marginal flow and stock damages will differ by an amount $\theta(t)$, which is constant in real terms:

$$-\frac{\partial C_i^+}{\partial E_i} - \left[\frac{\partial D^+}{\partial E} + f_S^+(t)\right] = -e^{\rho t}\theta(0) \qquad (43)$$

Not surprisingly, the addition of the single integral constraint over total emissions adds only one distinct term to the optimality condition, $\theta(0)$, the real shadow cost of emitting one more unit of the pollutant, at any time in the planning horizon. Clearly the sign of the shadow cost depends on whether the negotiated cumulative quantity of total emissions $\bar{E}_c$ is above or below the socially optimal total E_c^*.[17]

Table 1: Negotiated Emissions and Socially Optimal Cost			
Negotiated total emissions	$\bar{E}_c$ **vs** E_c^*	**Sign of θ(t)**	**Abatement Cost vs Flow and Stock Damages**
Too high	$\bar{E}_c > E_c^*$	$\theta(0) > 0$	$C_a < D_E + f_S$
Too low	$\bar{E}_c < E_c^*$	$\theta(0) < 0$	$C_a > D_E + f_S$

When the negotiated cumulative emission limit exceeds the socially optimal level of emissions, (i.e., the emission standard is too weak), the second-best permit reallocation leads to marginal abatement costs that are less than current and discounted future marginal damages. When the

[17]It is possible in principle for the negotiated level of emissions to be set at the dynamic social optimum, but this is unlikely.

socially optimal level of emissions is greater than the negotiated cumulative level of emissions, (i.e., negotiations over-restrict emissions), marginal abatement costs are greater than marginal flow damages plus the discounted future marginal stock damages. It is important to bear in mind that the future marginal stock damages, given a negotiated level of emissions, will be differ from the social optimum level of future damages. To make this type of "second best" system operational the environmental regulator needs to be able to estimate the aggregate abatement costs and aggregate damages through time *and* have the regulatory authority to reallocate emissions through time.

Suppose the regulatory authority would like to induce firms to control the time-sequence of their emissions in a nearly second-best optimal manner, given the negotiated emission permit levels for each period. In the absence of authority to directly reallocate emissions across time, one approach is to set up a banking system with an appropriate second-best ITR.

Allowing private banking and borrowing with a unitary (1:1) ITR leaves the cumulative sum of emissions unchanged, but in general will not attain the second best permit reallocation path. This is because private agents will arbitrage permits across time, until their price (and, therefor, marginal abatement cost) rises at the discount rate. On the other and, banking and borrowing with any *non-unitary* ITR will adjust the time path of permit use and abatement costs, but also can alter the total effective number of permits, which is potentially suboptimal. A non-unitary ITR allows the creation or destruction of permits through permit saving and borrowing and the accrual or payment of "interest." Of course, the total number of permits originally negotiated may be suboptimal, so this could be a desirable feature. Using the wrong ITR, however, can lower social welfare compared to a unitary ITR or the no-banking cases. The questions we seek to answer are: How can the environmental regulator use an ITR with banking and borrowing to improve net social welfare, and what are the magnitudes of the potential gains?

Exceeding the Second-Best Permit Reallocation

As mentioned above, a non-unitary ITR alters the cumulative sum of abatement. This raises the possibility that a non-unitary ITR can improve on the second-best emission path by attaining a net lower social cost of emission abatement and damages. That is, by choosing a non-unitary ITR, the environmental regulator can, in effect, compensate for the non-optimal negotiated emission standard.[18]

To find an efficient non-unitary ITR the environmental regulator must recognize how changing the banking rates $r_e(t)$ will influence the behavior of private agents, given their ability to choose when to bank and when to borrow and thereby determine the time evolution of marginal abatement costs. Thus, for the environmental regulator to choose the banking rate $r_E(t)$ in a

[18]Of course, the ITR may also be set by negotiation, in this case, there is little regulators can do ex-post to affect social welfare, unless the implied property rights of future emission allowances are renegotiated or the regulatory body is prepared to buy or sell emissions from its own account.

manner that maximizes the social objective, while accounting for the effect of banking on private agent behavior, the social planning problem must be augmented by a dynamic constraint that takes into account the firm's first-order conditions for the time rate of change of marginal abatement cost of the firm: $\hat{C}_a = i - r_E^{**}$. The environmental regulator's banking design problem must also account for the way in which non-unitary ITRs (nonzero banking interest rates) can alter the effective total number of permits issued. This can effectively be accomplished in the social planning problem by including the same bank account evolution equation and the positivity constraint on terminal bank balances that applies to the private problem (Eq. 21).

Comparisons of Solutions

We have now identified six ways to pursue the social objective of controlling the emission of a stock pollutant: the unconstrained social optimum, i.e., the 1st-best solution; a sub-optimal negotiated time stream of permits with no banking; a social "second-best" emission reallocation given the negotiated emission limits; a 2nd-best banking solution given a unitary ITR, still maintaining the cumulative emission limit; a banking solution using a non-unitary ITR with a single fixed value over the time horizon; and finally; a banking solution using a time-varying non-unitary ITR. This final approach allows both a more flexible control of the time path of marginal abatement effort and may entail a revision of the cumulative emission level. As such, we expect it to outperform more restrictive cases of unitary and single-valued ITR, and to potentially surpass the net social benefits of social 2nd-best reallocation.

Each approach entails a distinct set of emissions paths, permit prices, rates of growth in permit prices, and banking interest rates. The questions that need to be addressed are: which of these systems can be implemented via a permit banking system imposed on private agents; what information is required; and finally, what are the relative benefits of the various systems.

Three of the six social planning solutions described above may be implemented via a permit banking system placed on disaggregate private agents. In general, the social 1st best solution can be approached, but not fully attained by superimposing a banking system upon a suboptimal time-allocation of permits. Similarly, a private replication of the social 2nd best reallocation solution cannot be reproduced via banking. This is because there is no way, using unitary exchange rates, to force private agents to time-allocate emissions in a socially optimal way. Furthermore, there is generally no way, using non-unitary ITRs, to assure an unchanged level of cumulative emissions. However, we anticipate that the banking solutions with more flexible banking rates can approach, and possibly improve upon, the *net benefits* achievable under the social second best reallocation.

If the environmental regulator is able to estimate aggregate (across all sources) marginal abatement costs and aggregate (across all regions) marginal damages in each future period, then each of the above systems may be implemented.[19] Indeed, it is through the use of estimated aggregate cost and damage functions that we perform our numerical simulations described below. Acting without the aid of knowledge of future marginal damages and abatement costs places the environmental regulator in a difficult position. However, this burden is no less for case of regulator wishing to design a non-banking permit system and allocate permits.

In this paper two principal regulatory regimes are discussed: emission permits granted over time without banking or borrowing; and emission permits with banking and borrowing allowed under an intertemporal trading rate. The former provides greater flexibility and control to regulators, while the latter offers greater flexibility to firms. It may be preferable for the environmental regulator to *not* allow banking and borrowing, but instead to set the optimal amount of emissions at every point in time. To set optimally (at time 0) the intertemporal exchange rates for all periods in the planning horizon requires the same information as setting optimally each future time period's emission limits. Furthermore, regulators may prefer to prohibit banking and borrowing, because the continuous granting of annual permits allows them to revise permit allocations at a future time period, if deemed necessary. With banking and borrowing, polluters can acquire (buy and sell) property rights to emissions in the future.

If the regulator does not commit to future specified emission allocations, then the regulator's control and flexibility are unquestionably greater. At the same time, this imposes costs on polluters by limiting their ability to plan. If the regulator at time 0 does commit to future (non-bankable) emission allowances, then this would establish a property right just as in the case where polluters are allowed to bank and borrow emissions.

We are interested in examining when an environmental regulator can use non-unitary intertemporal exchange rates to coax firms to voluntarily emit a socially more desirable quantity of GHGs. We have not formally justified the need for an intertemporal banking and borrowing system with an exchange rate as opposed to simply setting the optimal allocation of permits in future time periods. Investigating intertemporal banking and borrowing can still be useful since: (1) it is politically popular (in the USA) and is under current consideration pursuant to Kyoto; (2) such a system provides the maximum flexibility to firms and nations in meeting their targeted emission reductions; and (3) allowing intertemporal trading of emission permits, regardless of whether contemporaneous permit trading occurs among firms, is a source of cost savings unless the permit allocation is optimal across time.

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[19]Incidentally, it is worth pointing out that implementing a carbon tax regime to attain the negotiated permit levels requires being able to estimate future marginal abatement costs, and attaining any of the first or second-best systems also requires being able to estimate future aggregate marginal damages.

9.0 Numerical Estimation of Single-Region Flexible GHG Banking Systems

We fully recognize the limits imposed by the simplifications necessary to compactly express global permit trading. At the same time we feel that intertemporal permit trading is one of the most powerful tools available to help control GHGs. Numerical estimation of flexible GHG emission systems requires that we adapt the theory described above to a discrete-time exposition. Appendix 11 discusses some of the key issues involved in this process and gives our solutions. For a simple adaptation of our theory we need to estimate a number of parameters and functions. In particular the we need estimates of the decay rate γ and the public and private discount rates. The residence time of CO_2 in the atmosphere depends on the rates of various biological and geophysical sinks (Trenberth 1992:218), and is sometimes represented by a detailed model rather than a fixed rate (Houghton *et al*., 1996:121). As an approximation, we can turn to the figure used by Nordhaus (1994:192, 1996), a decay factor of 8.33%/decade, or 0.8% per year.[20] For an exposition of the difficulties in establishing discount rates we can turn to the work of the Intergovernmental Panel on Climate Change (Arrow et al. 1996:131-133). Their balanced review of the literature presents rates for high-income industrial countries and also for developing countries. They find that equities have yielded a real rate of return of 5%, after accounting for taxation, or 7% pre-tax for many decades. The private (producer) discount rate would be expected to be at this pretax level, or possibly much higher, for some projects.[21] Commonly used estimates of the social discount rate (social rate of time preference) reported in Arrow et al. range between 1% and 3%. This implies, therefore, that r_e^* could be in the range of 3%-5% per year, even when current marginal stock damages are essentially zero. This "lower bound banking rate only accounts for the possible differences between public and private discount rates, and the decay rate of GHG stocks.

In Appendix 11 we display some computer code using the programming language GAMS (General Algebraic Modeling System) to estimate a banking and borrowing permit system based on the work of Falk and Mendelsohn (1993). We also show modifications to the code for the MERGE 2 and MERGE 4, (Manne and Richels 1992, Manne, Mendelsohn and Richels 1995), and DICE/RICE (Nordhaus 1994, Nordhaus and Yang 1996, Nordhaus and Boyer 2000).

Our intent, as specified above in the section on research objectives, is to apply the theoretical construct developed here to the existing numerical models. This will allow us to do three things:

Numerically validate our analytical insights;

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[20]This corresponds to a lifetime of 120 years. For greenhouse gas analysis, the lifetime is defined as the period over which the gas concentration falls to 1/e of its initial level. That is the "e-folding time," see Nordhaus, p. 26.

[21]For GHG banking, a key issue is which entities are allowed to make the permit borrowing/banking decisions (nation states, emitting firms, or speculators and traders), since that will have some bearing on the appropriate "private" discount rate for the analysis. This issue is taken up more in more detail in the multi-region models discussed below.

Numerically explore the magnitude of efficient banking rates and potential gains from banking;
Extend the analysis of permit banking numerically for more complex cases where our analytic treatment becomes intractable.

Although may others have thought about GHG permit trading we feel that the concepts we have developed on intertemporal trading ratios and our mathematical and computer skills places us in an excellent position to examine a whole series of questions relative to intertemporal trading of GHG permits.

In this section we assess numerically the potential relative benefits of the various banking systems. Using numerical estimates of marginal abatement costs and damages from Falk and Mendelsohn (1993)[22] we implement a discrete-time version of each of the banking systems discussed in the previous section using GAMS (General Algebraic Modeling System). Our numerical estimates use the same cost functions and damage functions given in Falk and Mendelson (1993). We generally use the same parameters, but diverge on our choice of discount and emission growth rates. In particular, for the low, medium and high discount rate cases we use social rates of 1%, 2%, and 4% respectively. We consider the possibility that private discount rates may exceed the social ones, and use 1%, 5% and 7%. We consider 1% and 2% growth rates for unabated emissions in our low and high cases. In addition there are low and high growth rates for abatement costs and environmental damages. Our results are not in any way intended to be definitive, but serve to illustrate the potential benefits of the various banking systems. Further work is clearly needed to better refine the numerical estimates.

We first calculate the 1st -best solution to the environmental regulator's problem over a 150 year time horizon. We then solve the private agent's problem to confirm that decentralized agents, given the optimal permit endowment and ITRs, will indeed achieve the 1st-best socially optimal solution. Next, we calculate the social welfare costs associated with a Kyoto-like negotiated emission constraint, *with no banking*, by solving the social problem with emissions limited to 1990 levels from 1990 through 2140. We also determine the extent to which the 1st-best emissions are greater or less than the standard, depending on the sensitivity case. For each of the three banking regimes discussed above (unitary ITR, fixed ITR, and variable ITR), we solve the appropriate social planning problem given that negotiated annual permit allocations are equal to the 1990 emission level. As a check, we implemented the private solution via the appropriate allocation of permits and ITRs, and assured that it behaves as expected. We then assess the social welfare costs of the various private banking regimes.

Table 2 shows the percentage reductions in social welfare for the social second best reallocation and the various banking systems compared to the 1st-best social optimum. Positive numbers indicate that the various alternatives are more costly and, as expected, inferior to the 1st-best

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[22]Ongoing additional work uses other models, including those by Manne and Richels (1997), Nordhaus (1994), Nordhaus and Yang (1996), and Nordhaus and Boyer (2000).

solution. What is interesting in looking at these numbers is the wide range of cost estimates of the various emission control and banking systems.

Given our admittedly rough damage and cost estimates, the first column shows the cost of meeting the negotiated emission standard when banking and borrowing are not allowed as compared to the socially optimal quantity and time allocation of emissions. These costs range from about 1% to 316% above the optimal quantity and time allocation of emissions. The costs are lowest when either emissions grow slowly and damages are low, or when emissions grow quickly and damages are high. Conversely, the costs are greatest when emissions grow quickly and damages are relatively low. Clearly, the likely evolution of unabated emissions and the susceptibility of society to damages are strong determinants of the costs of a negotiated standard which fixes emissions at current levels.

Table 2: Percent Reduction in Social Welfare from Various Banking Systems,* Compared to 1st-Best Social Solution							
Damages	Emissions Growth	Discount Rate	Case				
			Private No-Banking**	Social 2nd-Best Reallocation	Private Unitary 1:1 Banking	Private Single ITR Banking	Private Multi-ITR Banking
L	L	L	1.7	1.1	1.3	1.2	1.1
		M	18.5	13.3	21.3	2.8	1.0
		H	53.6	18.8	22.4	5.4	1.4
	H	L	65.7	60.3	60.3	57.3	30.4
		M	141.3	136.5	148.4	8.6	2.4
		H	315.8	232.0	242.2	12.1	3.4
H	L	L	98.9	96.2	100.7	100.3	98.1
		M	32.4	27.7	45.4	37.5	17.8
		H	2.4	1.6	10.2	4.2	0.5
	H	L	13.3	9.8	10.4	10.1	9.5
		M	1.3	0.1	8.9	4.4	1.1
		H	9.2	4.0	9.6	1.2	0.6

*In all cases except the 1st-best solution, the emission permits are initially allocated to keep emissions constant at the 1990 level.
**The private, no-banking case has the same social costs as the social problem subject to the negotiated emission limits.

Comparing the "social 2nd -best" reallocation case, column 2 in Table 2, to the no-banking case shows us the potential gains from the efficient time reallocation of the same quantity of emissions over the planning horizon of 150 years. As is seen, these potential cost reductions can be small (as in the low emissions growth, high damages case) or substantial (as in the low damage, high discount rate cases). As mentioned above, this 2nd-best optimal reallocation is not attainable through a bankable permit system. Nonetheless, it does provide an interesting benchmark.

The third column shows the percentage additional costs of implementing a unitary (1:1) ITR banking system given the negotiated permit allocations. Comparing column 3 to column 1 gives us the net percentage costs of allowing this form of naive banking and borrowing. As can be seen the results are mixed. In some cases the net social costs fall, and in other situations they rise. Again, if it were possible to better refine the future growth paths of costs and damages, then the desirability of allowing unitary banking would be clearer.

Turning attention to our efficient banking systems, using single-valued and multiple-valued ITRs, we observe that very significant cost savings are possible compared with the no-banking negotiated permit system. Indeed, as can be seen in columns 4 and 5, the non-unitary ITRs are often able to surpass the performance of the social 2nd-best reallocation.

Table 3 shows the differences in cumulative emissions between the various banking systems and the 1st-best social optimum. We observe that the differences in emissions between the no-banking case and the 1st-best can be positive or negative, as expected, depending on whether the emission standard is too weak or too strict. Not surprisingly, the amount by which the cumulative emissions diverge from the 1st-best generally corresponds to how much the net costs diverge. That is, the net costs are higher when cumulative emissions given no-banking, are furthest from the social optimum.

Table 3: Difference in Cumulative Emissions Between Private Banking Systems and 1st-Best Social Solution					
Damages	Emissions Growth	Discount Rate	Case		
			Private No-Banking*	Private Single ITR Banking	Private Multi-ITR Banking
L	L	L	-201	-198	-198
		M	-698	-34	-7
		H	-1092	461	28
	H	L	-3273	-3175	10
		M	-4498	36	32
		H	-5371	1017	115
H	L	L	1193	1194	1186
		M	970	1012	548
		H	538	664	69
	H	L	965	933	925
		M	94	604	136
		H	-1265	-49	-26

*In all cases except the 1st-best solution, the emission permits are initially allocated to keep emissions constant at the 1990 level, for a total of 7.92 bill MT CO2 equivalent/year or 1195.92 bill MT over the full 150 year time horizon. Cumulative unabated emissions would be 2279 and 7642 bill MT in the Low and High emissions growth rate cases, respectively.
**Negative numbers indicate that 1st-best emissions exceed the 1990 emission standard. Positive numbers indicate that the standard allows greater emissions than the 1st-best social optimal. .

It is important to remember that since the more efficient banking systems do not have a unitary exchange rate, they do not achieve the same level of emission abatement as the no banking or social 2nd best reallocation cases we have examined. They specifically answer the question of how an environmental regulator can reduce the costs of annual negotiated emission limit by allowing the non-unitary trading across time of emission permits. For scenarios in which the 1st-best solution is to allow greater emissions than those permitted under the negotiated standard (e.g., all low damages cases), the net result of using these non-unitary ITRs is to achieve both a different time path *and* a greater cumulative quantity of emissions. Conversely, for cases in which the standard is not sufficiently stringent (most high damages cases), these ITRs produce fewer net emissions than the negotiated limit.

10.0 Multi-Region Models

The theory and numerical simulations presented above all assume that the world can be represented by a single region, with a single environmental regulator making decisions for the world as a whole. This simplification allowed us to easily and readily characterize a second-best optimal intertemporal trading ratio to maximize net social welfare. In addition, this single-region characterization is useful for discussing *aggregate* emission, banking and borrowing behavior. In one sense, if all agents or regions can freely trade emission permits withing the same time period, then what really matters for intertemporal permit behavior, is the aggregate borrowing and banking behavior of all agents.

On the other hand, when moving to a multi-region characterization of the problem, it is no longer satisfactory to assume that a single entity can make decisions to maximize the net social benefits of GHG control in all regions simultaneously. That is, unless every region on the world has identical, homothetic preferences. To examine the more interesting situation where different regions of the world have different preferences for environmental quality we follow the lead of Rutherford (1992), and Manne and Rutherford (1994) and examine Pareto-optimal solutions to intertemporal equilibrium models. They rely on results of Chipman (1974) and Eisenberg (1961) who show that if individual agents have fixed endowments and (non-identical) homogeneous utility functions, then the aggregate demand function of the agents may be thought of as the result of the maximization of an appropriately-stated aggregate utility function, subject to the constraint that total expenditures equal total income.

The practical result is that we can recast global intertemporal permit trading (banking and borrowing) in a regional framework with interregional permit trade as well. In that framework each region or nation is viewed as a independent, price-taking agent maximizing its own well-being subject to its initial endowment of wealth. In particular the regional models that we use (described below) maximize the weighted and discounted sum of the logarithm of consumption. That is, global welfare for regions $r \in R$, Negishi weights N_r, and utility discount factor ρ_{rt} is given by:

$$\text{Global welfare} = \sum_{r \in R} N_r \sum_{t \in T} \rho_{rt} LN(C_{rt})$$

The equilibrium values of the Negishi weights correspond to each region's share of global resource endowments, evaluated at equilibrium prices. Maximizing a social welfare function defined in this way does not yield a social welfare maximum in the same sense that we were using the term earlier. This is because the equilibrium solution time paths of consumption and abatement reflect each region's share of global wealth endowments. Thus, an unequal or unfair distribution of initial endowments of labor, capital, *and emission permits*, ensures that, after taking into account intertemporal emission abatement, there will be an unequal or unfair final distribution of consumption and abatement. Thus, there is no unique social welfare maximum, since a redistribution of global endowments among regions (including permits) will induce a different global equilibrium. The Pareto-optimal solution will, however, be efficient in the sense that no other equilibrium could improve social welfare and be supported by market determined prices.

In this context, the first-best *aggregate* emission path across time is one that is collectively chosen by the regions taking into account market and non-market damages, and the initial distribution of wealth. Similar to our nomenclature for the single-region problem, we define the Pareto-optimal "second-best" reallocation of emission abatement path to be one that maximizes the aggregate social welfare while preserving the same cumulative emissions over time as those allowed under the negotiated emissions permits. This second-best emission path is the one that an environmental planner would choose given that the planner were able reallocate emissions and damages across time, changing *when* emissions occur while preserving the cumulative emission target. A unitary (1:1) banking system with unlimited banking and borrowing (subject to non-negativity constraint on the terminal period bank account) generates such a second-best solution. Given a negotiated emission path over time (not necessarily identical to the Pareto-optimal allocation) it is useful to ask how an intertemporal permit trading system, with non-unitary exchange rates can improve aggregate social welfare, compared to not allowing intertemporal banking and borrowing at all, or compared to allowing banking and borrowing at a unitary exchange rate.

MERGE

The first model that we adapt for our regional intertemporal exchange rate is MERGE (a Model for Evaluating the Regional and Global Effects of greenhouse reduction policies) by Manne and Richels which is available in different versions (1992, 1995, 2000).[23] In MERGE2 there are five regions: United States, other OECD nations (Western Europe, Japan, Canada, Australia, and New Zealand), former Soviet Union, China and the rest of the world. Each region is viewed as an independent price taking agent and is subject to an intertemporal budget constraint. At each point in time supplies and demand are equilibrated through the price of internationally traded commodities: oil, gas, coal, carbon emissions rights and a numeraire good.

A number of changes to the MERGE 2 model were necessary in order to examine intertemporal banking and borrowing. These include an equation of motion for banking and borrowing permits, a terminal condition for the permit bank account, and a number of less obvious, but necessary changes. The changes to the MERGE 2 code are shown in the Appendix.

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[23]MERGE versions 2-4 are generously made available by the authors and EPRI by license, see http://www.stanford.edu/group/MERGE/.

Proceeding as in the single region model, implementing the social 2nd best problem requires that each region properly anticipate the effect of a banking system with non-zero intertemporal exchange rates on the time path of marginal abatement cost and on the quantity of total allowable emissions. Since the MERGE 2 model does not have an explicit expression for marginal abatement costs, the current value price of permits can be used as a proxy for current value abatement costs. Thus, our first experiments involved adding a restriction on the time rate of change of permit prices via choosing the optimal IE rate, $i_e(t)$.

$$\hat{p}_{et} = \frac{p_{t+1} - p_t}{p_t} = \frac{1 + \rho(t)}{1 + i_e(t+1)} - 1 \tag{46}$$

As it turns out, this procedure, which conceptually correct, is not practicable in the MERGE framework. This is because permit prices, as determined in the original MERGE code, are a dual variable on the permit trade balance equation that (along with the rest of the model) is solved iteratively. In addition, the marginal productivity of capital, is a parameter. Therefore, the model, when trying to endogenously determine the optimal second-best intertemporal trading rate, always runs into an infeasible constraint since there is insufficient feedback through prices to affect a feasible solution. Attempts to circumvent this failure via the addition of a penalty-weighted slack variable were successful in generating feasible solutions, but never without a significant inclusion the slack variable in the solution set. We conclude, therefore, that the structure of the MERGE solution algorithm: a primal-dual, iterative method, is incompatible with the direct application of our theory.

Fortunately, we can apply our theory indirectly. In particular, we can introduce the endogenously determined intertemporal exchange rate, IE_t, in the equation of motion for permits. Using the notation defined above, with the addition of $\bar{E}_{t,r}$ representing the allocation of permits to region r in period t, and $X_{t,r}$ denoting net permit exports, we get the following regional equation of motion for the emission permit bank balance $B_{t,r}$:

$$B_{t+1,r} = (\bar{E}_{t+1,r} - E_{t+1,r}) - X_{t+1,r} + B_{t,r} * (1 + IE_t) \quad \forall\ t<T, r \in R \tag{47}$$

This formulation treats B_t as the end of period bank account. The initial period bank balance is zero. A terminal period constraint forces regions to have non-negative bank balances at the end of the regulatory period.

$$B_{t=T,r} \geq 0 \quad \forall\ r \in R$$

Regional Issues

In moving to a regional framework there are some additional issues that occur compared to the single-region model. One of these is whether to allow region-specific intertemporal IE_t rates. One could argue that of course intertemporal trading rates should be set independently for each region, just as endowments are regionally set. Given unrestricted permit trading among regions, however, unequal IE rates across regions cannot be sustained in the face of inter-regional arbitrage. Regional trade equalizes permit prices across regions, while unequal IE rates would entail unequal growth rates of permit prices across regions, a conflict.

A second regional permit trading issue that arises is that of the indeterminacy of regional permit trading and banking. That is, when regions can bank and borrow emissions across time and freely buy and sell permits amongst themselves, they are indifferent between banking and exporting, and indifferent between borrowing and importing. So in the absence of transaction costs there are many equivalent combinations of banking and trading, from the perspective of each region. The intertemporal permit price is determined by the *total* number of permits allocated to all regions and the equalized global opportunity cost of abatement. Simply stated, with free trading among regions, emission banking is a system-wide phenomena. In formal models like our modified version of MERGE, regions that maximize intertemporal consumption are indifferent between borrowing, banking or trading a permit in each particular time period. The present value price of permits changes through time reflecting financial discounting, changes in abatement technology (energy efficiency) and current and future marginal damages, and other factors. Thus, while the sum of permits banked or borrowed across all regions is well-defined in any time period, each individual region's banking and borrowing behavior is not. This result clearly only pertains to a stylized world absent transaction costs and absent any uncertainty about future repayment of debts or changes in regulatory policy. To eliminate this indeterminacy between trading and banking, we introduce a very small transaction cost to permit sales between two regions.

To investigate the potential benefits of intertemporal permit exchange rates we compare the results of each region maximizing its own welfare under different trading regimes and different assumptions about how damages from global warming enter into the regional decision calculus. We run the model with each region taking into account market and non-market damages, allowing unitary (1:1) banking and borrowing, and permit trading. We then compare this solution to one where each region ignores market or nonmarket damages or both. We can also compare these solutions to the unrestricted Pareto-optimal outcome, no emission constraint, from the original MERGE formulation. Next, we can run these same experiments, but allow for the endogenous selection of an intertemporal exchange rate for permits. These experiments tell us the potential gains under a variety of scenarios concerning how regions view damages, from the use of non-unitary intertemporal exchange rates.

The objective function in the MERGE model for accounts for non-market and market damages. In the Pareto-optimal case, non-market damages enter the Negishi welfare definition, and market damages represent one of the competing claims on the allocation of total production resources, or output. In the Pareto-optimal, Negishi welfare function shown below, non-market damages are reflected in the prospect for catastrophic loss. The parameter $CATT_r$ defines the region-specific value of the catastrophic level of temperature. The parameter HSX modifies the quadratic loss function so as to allow for a hockey-stick shape.

$$\text{Negishi Welfare} = 1000 * \sum_{r \in rg} N_r \cdot \sum_{t \in tp} udf_{r,t} \cdot LN\left(C_{r,t} \cdot \left(1 - \left(\frac{ATP_t}{CATT_r} \right)^2 \right)^{HSX_{r,t}} \right) \tag{49}$$

where

ATP_t is actual temperature,
$CATT_r$ is catastrophic temperature,
$udf_{t,r} = 1 - MPC_t + GROW_{t,r}$

Competing claims on output $Y_{r,t}$ include consumption, investment, energy costs, market damages and net exports of the composite numeraire good:

$$Y_{t,r} = C_{t,r} + I_{t,r} + EC_{t,r} + MD_{t,r} + NTX_{t,r}(''nmr'') \quad \forall t \tag{50}$$

MERGE 2 Results

As a baseline for examining the impact of our intertemporal permit exchange rate we started with the MERGE 2 case that examined the costs of stabilizing global emissions (SGE) at 1990 emission levels around the world. These baseline emissions are given in the table below.

Table 3: 1990 Baseline Carbon Emissions[24]

Nation or Region	1990 Billion Metric Tons of Carbon
United States	1.417
Other OECD nations (Western Europe, Japan, Canada, Australia, and New Zealand)	1.485
Former Soviet Union	0.960
China	0.623
Rest of the world.	1.379
World	5.864

This is a very strict standard since it freezes the whole world's emission, not just those of Annex 1 nations. Nonetheless, it gives us insight into how an intertemporal permit exchange rate system could work, and is directly comparable to the published literature on MERGE (see for example: Manne, Mendelsohn and Richels, 1995). The cases we examine in detail as described in Table 4. For the simulations discussed below, we have further restricted the intertemporal exchange rate to be constant across time.

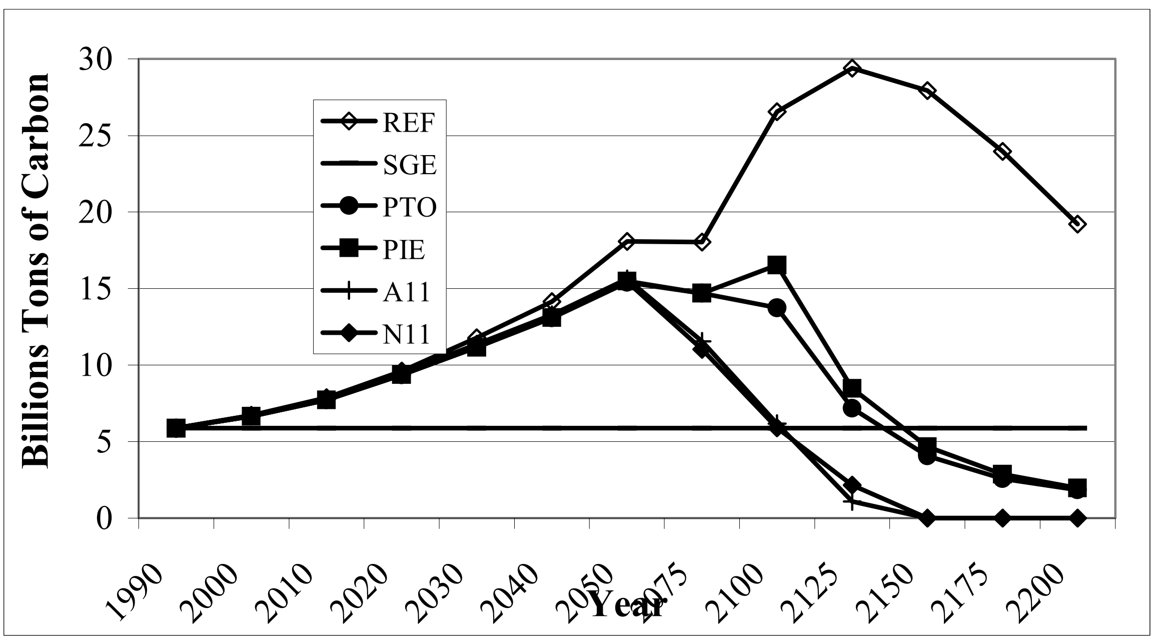
[24]Source: MERGE 2 code, Table CARLIM

Table 4: Policy Scenarios				
Scenario Name	Policy Description	Damages Considered in Choosing Emissions		Banking and Borrowing Allowed?
		Market?	Non-Market?	
REF	Reference case pertaining to the business as usual with no limit on emissions. Results the same as in the original MERGE 2 model	No	No	No (not applicable)
PTO	Pareto-optimal case, no emission limits, each region takes market damages and non-market damages into account. Results the same as in the original MERGE 2 model	Yes	Yes	No (not applicable)
SGE	Stabilize global emission, emission limits are place on each region equal to their 1990 emissions. Regions can trade, but not bank or borrow emissions. Results the same as in the original MERGE 2 model	No	No	No
P11	Regions consider all damages. Pareto-optimal permit banking and borrowing on 1:1 ratio across time	Yes	Yes	Yes, unitary (1:1)
N11	Regions ignore non-market damages, but consider market damages. Permit banking and borrowing on 1:1 ratio across time	Yes	No	Yes, unitary (1:1)
A11	Regions ignore all market and non-market damages, permit banking and borrowing on 1:1 ratio across time	No	No	Yes, unitary (1:1)
PIE	Regions consider market and non-market damages, but the intertemporal exchange rate (IE) is chosen endogenously to maximize world welfare	Yes	Yes	Yes, endogenously chosen inter-temporal exchange rate IE
FNIE	Regions ignore non-market damages, permit banking and borrowing occurs at the IE rate determined in the PIE case	Yes	No	Yes, at the IE rate determined in the PIE case
FAIE	Regions ignore all market and non-market damages, permit banking and borrowing occurs at the IE rate determined in the PIE case	No	No	Yes, at the IE rate determined in the PIE case

MERGE 2 Results for the Time Path of Emissions

We see in Figure 1 the global carbon emissions for the three reference scenarios (SGE, REF, and PTO), graphed against three banking scenarios.[25] In each banking case the initial allocation of permits corresponds to the emission levels in the 1990 stabilization case. In the unitary, or 1:1 banking cases, the regions collectively borrow emissions heavily in the early years and pay back in the later years. In fact, these two cases find it cost-effective to widely exceed the stabilization target in the early and middle years, in exchange for using very expensive ($1000 per ton) carbon-free technology to emit zero greenhouse gases in the later half of the 22nd century. Clearly, the long time horizon, with its attendant discounting and assumed autonomous decarbonization trend, accounts for this being the optimal strategy. This make perfect sense, however, since these two cases, by construction, partially or wholly ignore environmental damages. The Pareto-optimal case with no emission limits (PTO), and the Pareto-optimal case with banking (PIE), which both consider all damages, also choose emission paths much greater than the 1990 stabilization case (SGE). But these Pareto-optimal case emissions are still well below the unrestricted emissions case ignoring damages (REF).

Figure 1: Global Carbon Emissions



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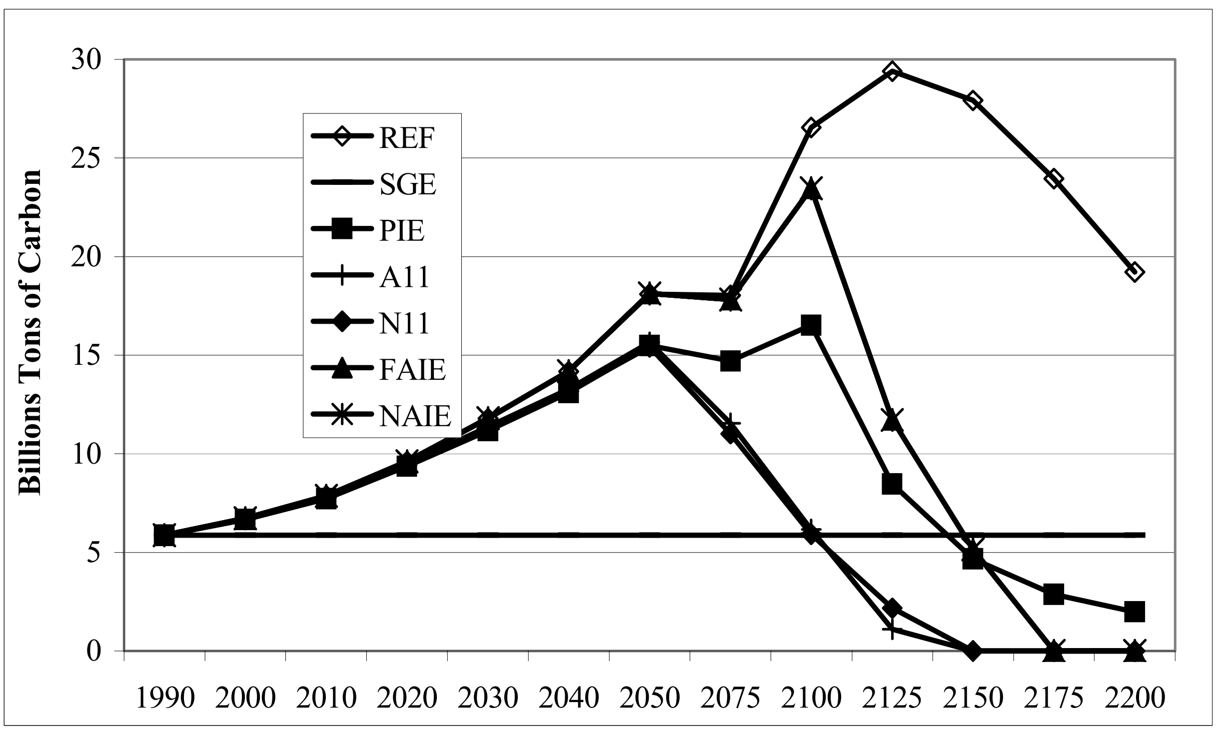
[25]The banking scenarios are for 1:1 banking assuming regions do not care about non-market damages (N11);1:1 banking assuming that regions do not take into consideration any (market or non-market) damages (A11); and the case where the trading ratio can be chosen (endogenous to the optimal solution) in order to maximize world welfare when both market and non-market damages are taken into account by each region (PIE).

The endogenous banking rate case (PIE) achieves a path close to the unrestricted Pareto-optimal case via the selection of an intertemporal trading (IE) rate of -1.5% per annum. That is, the PIE case finds that, given the very strict emission constraint of stabilizing world emissions at 1990 level, it is preferable, when taking damages and abatement costs into consideration, to allow regions to trade emissions across time at a 0.985:1 trading ratio. Stated otherwise, 0.985 permits now can be traded for 1 next year. The negative interest rate means that world welfare could be improved over the SGE permit baseline by discounting borrowed emissions in order to by relax the emission standard. This result is not an artifact of our banking and borrowing system or of our intertemporal exchange rate. It reflects the calibration of MERGE 2. Indeed, this result is foreshadowed by the reference case results that shows the PTO emissions greater than the SGE emissions. Indeed, not shown here is that the price of carbon permits under the SGE permit allocation is rising through time at a rate slower than the discount rate. That is, real permit prices are declining in the stabilization case SGE. This is because of MERGE 2 assumptions about the rate of energy use growth and rate of price-induced and autonomous energy efficiency improvements. In scenarios that decrease the emission permit allocation over time, the optimal solution is to bank emissions and choose a positive intertemporal interest rate.

Figure 2: Global Carbon Emissions



In Figure 2 we add two new emission paths to those shown in Figure 1; labeled FNIE and FAIE. These emission paths show how regions would choose (given that they do not take damages into account) to emit emissions in the face of the SGE permit allocation but subject to the IE rate chosen that maximize global welfare (including damages and abatement costs). As is seen these two emission paths are almost identical to one-another and are above the optimally chosen permit emission path (PIE), but significantly lower than the REF, business-as-usual path. The NAIE and FAIE paths are also above the emission paths for the 1:1 permit banking cases (A11 and N11). This is because the negative IE rate allows the NAIE and FAIE cases to emit more GHGs and improve social welfare.

MERGE 2 Results for the Time Path of Cumulative Costs and Damages

In Figure 3 cumulative costs (including abatement costs, market damages, non-market damages) are shown aggregated across all regions and discounted at 5% per annum. As alluded to above, the SGE case has by far the highest total costs, and the unconstrained Pareto-optimal case has the lowest costs. The REF, PIE and N11 and A11 cases fall in between these two benchmarks, but much closer to the PTO side.

Figure 3: Cumulative Damages and Abatement Costs, 5% Discount

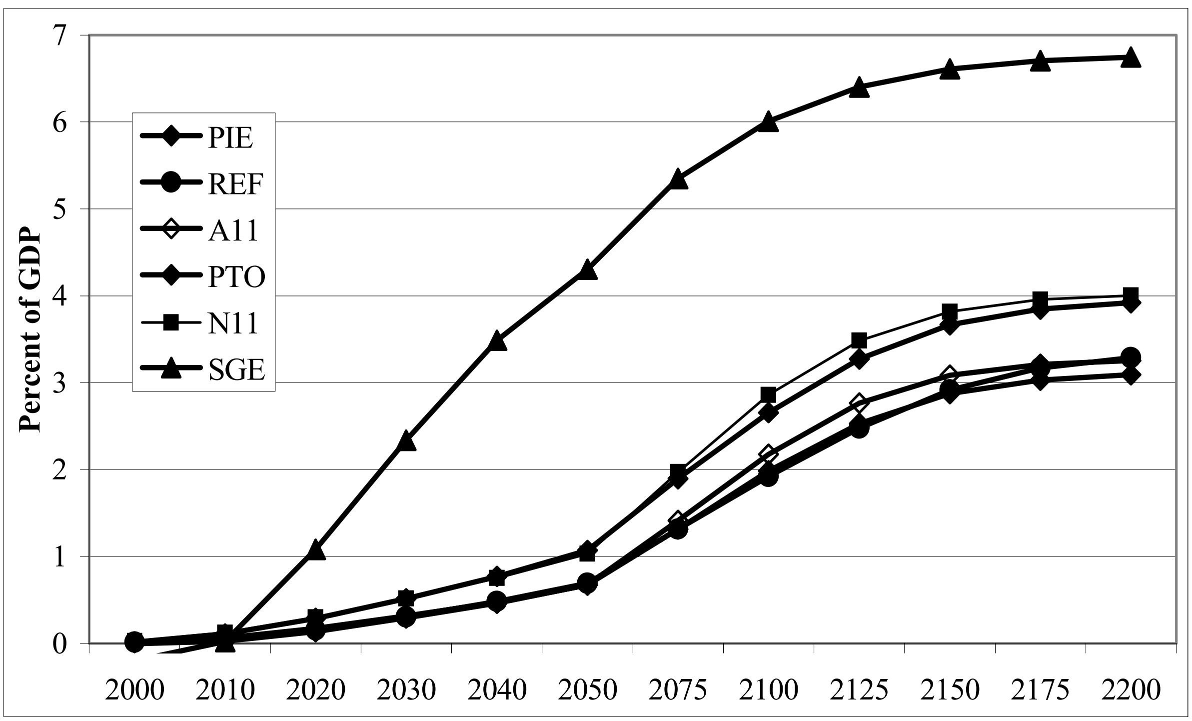


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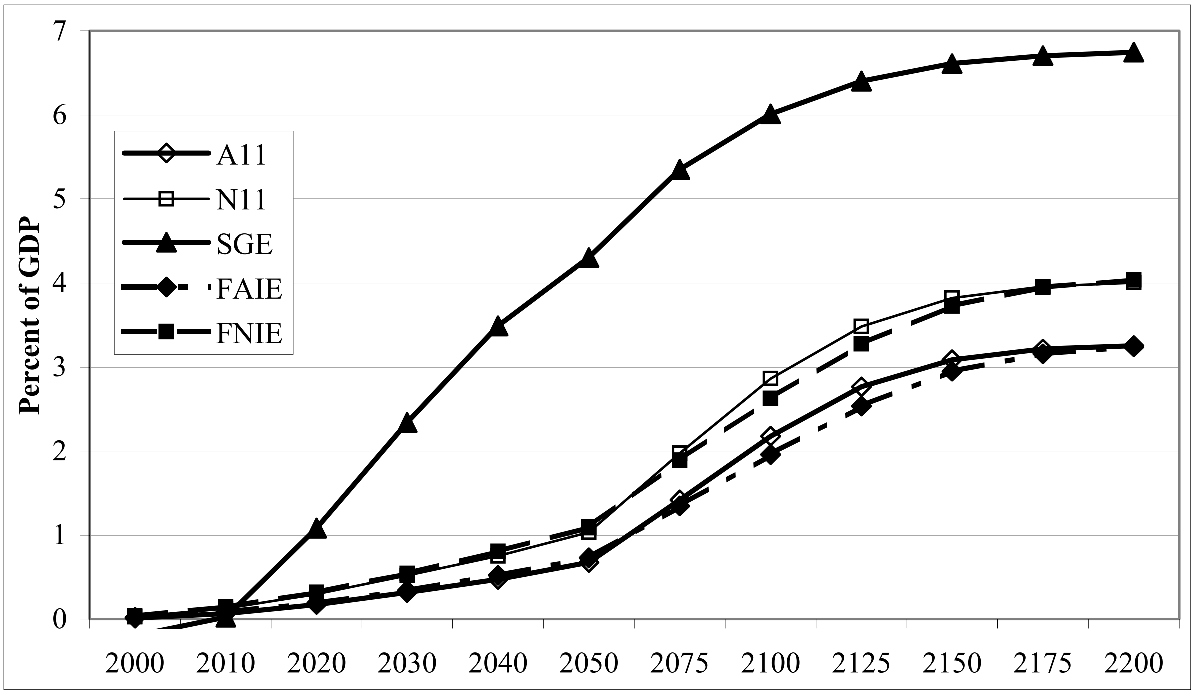
Figure 4: Cumulative Damages and Abatement Costs, 5% Discount



In Figure 4 we again show cumulative, discounted damages, but focus attention on how the optimally chosen IE rate affects social welfare. We thus, want to compare cumulative damages between the pairs of cases A11 and FAIE (all damages ignored) and N11 and FNIE (non-market damaged ignored). We have left the stabilization case SGE for reference. As is seen, by optimally choosing the IE rate to be -1.53%, we are able to induce independent regions *that ignore damages*, into decreasing the net costs to the world of GHG emissions and control. That is, by starting from an non-optimal emission profile, we show that it is possible to use a carefully chosen intertemporal emission exchange ratio to decrease net world costs of GHG control in a multi-region framework. The particular numerical result of an IE rate of -1.53% reflects the full range of cost, damage, and climate assumptions contained in MERGE 2 as well as our taking as a starting point allocation of emission permits based on each region's 1990 GHG emissions.

11.0 Final Comments

Permit banking and borrowing represent a promising tool to achieve environmental standards while allowing firms, or nations in the case of greenhouse gas emissions, a great deal of flexibility in how and when to fulfil their obligations. Implementing a permit banking system allows firms to shift emissions through time. This alters the timing of damages, as well as the magnitude of marginal damages. With stock pollutants, the timing of emissions and damages is no longer coincident. This does not mean that banking and borrowing should be discouraged. On the contrary, permit banking and borrowing in the presence of a known set of permit endowments can provide important flexibility to firms and nations. But, like any policy tool, banking needs to be implemented correctly.

In contrast to single-period tradable permits, allowing firms to bank or borrow permits on a one-to-one basis will not generally produce a socially efficient allocation of abatement efforts. Furthermore, in the face of a non-optimal time sequence of permit endowments, prohibiting banking and borrowing will also not be socially optimal. With permit banking and borrowing, regulators have two policy tools to work with: the total allocation of permits and the banking and borrowing rate of for intertemporal exchanges. The first tool sets the overall level or present value of discounted permit prices; the second tool adjusts the time rate of change of permit prices and, for non-unitary ITRs, the total allowable emissions.

This paper derives the cooperative, socially optimal permit banking and borrowing system for a dual permit system for a pollutant that creates both stock and flow damages. This system extends and unifies the existing literature on intertemporal permit trading, which can now be seen as special cases of our generalized stock and flow pollutant model. As an important special case we derive a flow permit system for a pollutant whose damages depend only on the accumulated pollutant stock. The latter case corresponds roughly to the greenhouse gas emission reduction regime recently proposed by the U.S. Department of State.

For greenhouse gas emissions such as CO_2, which have no associated flow damages, we show that the optimal banking interest rate is equal to the ratio of current marginal stock damages to the discounted future value of marginal stock damages less the decay rate of emissions in the atmosphere. If the “private” discount rate used by the individual agents who own and trade permits differs from the “social” or collective planning discount rate, then the intertemporal banking rate should be increased by that difference. Given an optimal endowment and allocation of permits across time, however, the optimal ITR will ensure that private agents do not bank or borrow permits.

Of particular current policy relevance, we show how an environmental regulator can use the permit banking and borrowing system to improve social welfare, given an non-optimal permit endowment path. To accomplish this, the environmental regulator must be able to estimate aggregate marginal damages and aggregate marginal costs over the relevant time horizon. If this is possible, then the environmental regulator can use a non-unitary ITR to partially compensate for the incorrectly chosen permit path. This case corresponds, for example, to an international permit system designed to limit GHG emissions when permit levels are fixed at a pre-determined level such as that negotiated under the Kyoto Protocols.

Finally, we use GHG abatement and damage cost estimates available in the published literature to examine the magnitudes of the expected gains from implementing the various permits systems examined. Our results are not in any way intended to be definitive, but serve to illustrate the potential benefits of the various banking systems. Our results show that a poorly designed banking system, one that naively allows 1 for 1 trading over time for example, can substantially raise the net social costs of GHG abatement. On the other hand, a permit system using a non-unitary ITR can, depending on the particular sensitivity case under analysis, provide substantial cost savings as compared to a permit system loosely based on the Kyoto protocol. A key determinant of the potential cost savings is the likely evolution of unabated emissions and damage sensitivity paths over time.

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Appendix 1: Discrete-Time Adaptations to Permit Banking and Borrowing

Discrete-Time Banking Model

In order to implement the theory described above, we need to translate key continuous-time model statements into discrete time rules that can be implemented using mathematical programming. In determining our key relationships, we ask which statement of the social problem reflects the imposition of a banking system on the market (accounts for the permit grow through interest), and thereby allows the determination of the social second best ITRs in the face of such a system? Alternatively, we can ask what alternative formulation of the private problem with banking yields the "Social 2nd Best" outcome, as currently defined (with no change in cumulative emissions over time from the negotiated path).

The Problem:
The social 2nd best problem and the private banking problem do not subject emissions to the same cumulative constraint, hence the solutions are unlikely to match, and may not even have overlapping feasible regions.

The current statement of the Social 2nd Best problem assumes that the cumulative (undiscounted) sum of emissions cannot change from the negotiated amount, but allows shifting emissions over time, i.e.

$$\begin{aligned} &\sum_{t=0}^{T} e_t \leq \sum_{t=0}^{T} \underline{e}_t \\ &\Leftrightarrow \\ &\sum_{t=0}^{T} \left(\underline{e}_t - e_t\right) \geq 0 \end{aligned} \tag{51}$$

In contrast, the private problem with banking is subject to the following constraints.

$$\begin{aligned} &\textit{BEEQ}: \\ &B_{t+1} = B_t(1+i(t)) + \underline{e}_t - e_t \\ &\textit{BEFINALEQ}: \\ &\qquad B_T + \underline{e}_T \geq e_T \\ &\textit{and} \\ &\qquad\quad B_0 = 0 \end{aligned} \tag{52}$$

Note that here the bank account B_t is defined as the *entering* balance in period *t*, and we can't really say whether interest is accrued during periods or between the end of one period and the start of another. Nonetheless, there is a conceptual error in the terminal constraint (which we foresaw, and which is small but nagging), in that it does not treat B_{T+1} in the same way as B_t for every other *t*. Assume first that we fix this as follows:

BEFINALEQ:

$$(1+i(T))B_T + \underline{e}_T \geq e_T$$

i.e.:

$$B_{T+1} \equiv (1+i(T))B_T + (\underline{e}_T - e_T) \geq 0 \qquad (53)$$

Then it is useful to write the recursive constraints on the bank account balances in summation form:

BEEQ:

$$\begin{aligned}
B_0 &= 0 \\
B_1 &= (1+i(0))0 + \left(\underline{e}_0 - e_0\right) \\
B_2 &= (1+i(1))\left(\underline{e}_0 - e_0\right) + \left(\underline{e}_1 - e_1\right) \\
B_3 &= (1+i(1))(1+i(2))\left(\underline{e}_0 - e_0\right) + (1+i(2))\left(\underline{e}_1 - e_1\right) + \left(\underline{e}_2 - e_2\right)
\end{aligned} \qquad (54)$$

and generally

$$B_{t+1} = \sum_{\tau=0}^{\tau=t}\left[\left(\underline{e}_\tau - e_\tau\right)\prod_{z=\tau+1}^{t}(1+i(z))\right], \quad for\ t \geq 1$$

Since only the terminal bank account balance is constrained, the private banking system reduces down to a single constraint (on B_{T+1}):

$$B_{T+1} \equiv \sum_{t=0}^{T}\left(\underline{e}_t - e_t\right)I(T,t) \geq 0$$

$$I(T,t) = \begin{cases} \prod_{\tau=t+1}^{T}(1+i(\tau)) & t<T \\ 1 & t=T \end{cases} \qquad (55)$$

The key point is that this constraint (55) on cumulative emissions does not match the social 2nd best constraint (1), and in fact the social 2nd best emissions path may not even be feasible under this constraint.

By the way, the multiperiod interest factors $I(t_2,t_1)$ really are what we should be referring when we speak of Intertemporal Trading Ratios (ITRs), rather than $i(t)$. The latter are only single-period interest rates, while the former factors $I(t_2,t_1)$ are indeed the actual ratios by which permits in one period (t_1) can be exchanged for another (t_2).

$$\frac{I(T,t)}{I(T,t+1)} = 1 + i(t+1) \qquad (56)$$

Solution to the Social 2nd Best Banking Problem:

While constraint on terminal bank account as written in cumulative form above (Eq. 55) is interesting, we now realize that it is not necessary for solving the Social 2nd Best (optimal banking) problem given negotiated permit allocations $\underline{e}$(t). Because it introduces the new ITR variables $I(T,t)$, which are multiplicative products of the Banking Rates $I(t)$ for many periods, solution of the model with Eq. 55 would also be also problematic.

All that is required, however, is that the Social 2nd Best problem properly anticipate the effect of a banking system with non-zero banking rates on the marginal abatement cost time path and on the total allowable emissions of private agents. To capture the effect of the banking system on total allowable private emissions, we can simply include the emission permit stream $\underline{e}(t)$ and the same banking equations faced by the private agents:

$$
\begin{aligned}
&BEEQ: \\
&\quad B_{t+1} = B_t(1+i(t)) + \underline{e}_t - e_t \\
&BEFINALEQ: \\
&\quad (1+i(T))B_T + \underline{e}_T \ge e_T \\
&and \\
&\qquad B_0 = 0
\end{aligned}
\tag{57}
$$

The banking rate $i_e(t)$ can be made an endogenous variable for the second best optimal banking rate problem. These constraints alone are not enough, however, because the Social Planner must also recognize how changing the banking rates $i_e(t)$ will also determine the time evolution of marginal abatement costs, hence abatement.

Effect of Banking Interest Rate on Marginal Abatement Cost Evolution in Discrete Time

The key issue is how does the discrete-time (DT) bank account condition translate to private abatement effort? Recall first the continuous time (CT) formulation of the problem:

CT Private Problem:

$$
\begin{aligned}
&\underset{a(t)}{Min} \int_0^T C(a,t)e^{-\rho_p t}dt \\
&s.t. \\
&\quad \dot{B} = (\underline{e}(t) - e_u(t) + a(t)) + i_e(t)B \\
&\qquad B_0 = 0, \quad B(T) \ge 0 \\
&\Leftrightarrow \\
&B(T) = \int_0^T (\underline{e}(t) - e_u(t) + a(t))e^{\int_t^T i_e(\tau)d\tau} dt \ge 0
\end{aligned}
\tag{58}
$$

Forming the continuous time Hamiltonian, H_{CT}, and applying the maximum principle:

$$
\begin{aligned}
H_{CT} &= C(a,t)e^{-\rho_p t} + \theta(t)\big((\underline{e}(t) - e_u(t) + a(t)) + i_e(t)B\big) \\
\frac{\partial H_{CT}}{\partial a} &= C_a(t)e^{-\rho_p t} + \theta(t) = 0 \\
\dot{\theta} &= -\frac{\partial H_{CT}}{\partial a} = -\theta i_e \quad \Rightarrow \quad \theta(t) = \theta(0)e^{-i_e t} \\
C_a(t) &= -\theta(0)e^{(\rho_p - i_e)t} \\
\hat{C}_a(t) &= (\rho_p - i_e)
\end{aligned}
\tag{59}
$$

The comparable Discrete Time (DT statement of the private agent's problem is:

DT Private Problem:

$$\underset{a_t, x_t}{Min} \sum_0^T \left(C_t(a_t) + px_t\right)\delta_p(t)$$

$$\delta_p(t) \equiv e^{-\rho_p t} dt$$

$$s.t.$$

$$B_{t+1} = (\underline{e}_t - e_{ut} + x_t + a_t) + (1 + i_{et})B_t \tag{60}$$

$$B_0 = 0, \quad B_{T+1} \geq 0$$

$$\Leftrightarrow$$

$$B_{T+1} = \sum_0^T (\underline{e}_t - e_{ut} + x_t + a_t)I(T,t) \geq 0$$

Writing the DT Lagrangian and applying the first order conditions:

$$L_{DT} = \sum_0^T \left(C_t(a_t) + p_t x_t\right)\delta_p(t) + \theta \sum_0^T (\underline{e}_t - e_{ut} + x + a_t)I(T,t)$$

$$\frac{\partial L_{DT}}{\partial a_t} = C_a(t)\delta_p(t) + \theta I(T,t) = 0$$

$$\frac{\partial L_{DT}}{\partial x_t} = p_t \delta_p(t) + \theta = 0 \tag{61}$$

$$\Rightarrow \quad C_a(t) = -\theta \frac{I(T,t)}{\delta_p(t)}$$

$$\Rightarrow \quad p = -\frac{\theta}{\delta_p}(t)$$

Here we see that the current value permit price is equal to the current value of the Lagrange multiplier on the emission constraint. This implies that the current value of a permit is equal to the value of relaxing the emission constraint. From this condition we can determine the expected relationship between the time-rate-of-change of marginal abatement cost and the Banking Rate $i_e(t)$. Note that there are a variety of ways to write the DT version of the rate of change of abatement cost, using different leads and lags, and that all are acceptable definitions, each implying a slightly different relationship between $\hat{C}_a$ and Banking Rate $i_e(t)$. In our modeling so far we have adopted the following lagged definition:

$$\hat{C}_{at} = \frac{C_{at} - C_{at-1}}{C_{at}} \tag{62}$$

Substituting in the optimality condition from the DT Private banking problem:

$$\hat{C}_{at} = \frac{\frac{I(T,t)}{\delta_p(t)} - \frac{I(T,t-1)}{\delta_p(t-1)}}{\frac{I(T,t)}{\delta_p(t)}} = 1 - \frac{I(T,t-1)}{I(T,t)}\frac{\delta_p(t)}{\delta_p(t-1)} \tag{63}$$

Using a "continuous" form for the discount factor in the model, with a discrete time step of Δ, the following relationship applies:

$$\delta(t) \equiv e^{\rho_p t\Delta} \quad \Rightarrow \quad \hat{C}_{at} = 1 - \frac{1+i_{et}}{e^{\rho_p \Delta t}} \tag{64}$$

Alternatively, if we used a discrete form for the discount factor in the model, with a discrete time step of Δ, the following relationship would apply:

$$\delta(t) \equiv \frac{1}{(1+\rho_p)^{t\Delta}} \quad \Rightarrow \quad \hat{C}_{at} = 1 - \frac{1+i_{et}}{1+\rho_p} = \frac{\rho_p - i_{et}}{1+\rho_p} \tag{65}$$

Thus, for the social 2nd best banking system to endogenously choose the banking rate i_e(t) in a manner that maximizes the social objective while accounting for the effect of banking of Private agent behavior, the social planning problem must be augmented by constraints imposing the banking equations of Eq.55, and by a constraint on marginal abatement cost of the form:

$$\text{PSCONEQ:} \quad \hat{C}_{at} = \frac{C_{at} - C_{at-1}}{C_{at}} = 1 - \frac{1+i_{et}}{e^{\rho_p \Delta t}} \tag{66}$$

Leading Formulation

In the MERGE 2 model, we now have the equation of motion for carbon credits written as a leading function:

$$CLEVD_{T+1} + NTXD_{T+1} + B_{T+1} = SHD_{T+1} * WCARBD_{T+1} + B_T * (1 + IE_T) \quad \forall \quad regions \tag{67}$$

This means that we need to recalculate the relationship between the rate of change in marginal abatement costs and the intertemporal exchange rate.

$$\hat{C}_{at} = \frac{C_{at+1} - C_{at}}{C_{at}} = \frac{\dfrac{I(T,t+1)}{\delta(t+1)} - \dfrac{I(T,t)}{\delta(t)}}{\dfrac{I(T,t)}{\delta(t)}} \tag{68}$$

$$\hat{C}_{at} = \frac{I(T,t+1)}{I(T,t)} \cdot \frac{\delta(t)}{\delta(t+1)} - 1 \tag{69}$$

Finally, this reduces to the following restriction on the rate of change in marginal abatement costs over time.

$$\hat{C}_{at} = \frac{1 + \rho}{1 + i_e(t+1)} - 1 = \frac{\rho - i_e(t+1)}{1 + i_e(t+1)} \tag{70}$$

Since the MERGE 2 model does not have an apparent expression for marginal abatement costs, the *current value* price of permits will be used. Therefore,

$$\hat{p}_{et} = \frac{p_{t+1} - p_t}{p_t} = \frac{1 + \rho}{1 + i_e(t+1)} - 1$$

$$(p_{t+1} - p_t)(1 + i_e(t+1)) = p_t(\rho - i_e(t+1)) \tag{71}$$

Appendix 2: GAMS Code for Single-Region GHG Banking Model

SUBSCRIPTS

t,τ	index over time periods
T	Final time period

SCALARS

E_0	Initial Gross Emissions	/7.92 /
S_0	Initial Pollutant Stock	/800 /
D_1	Damage Fn Linear Coeff	/-0.0325 /
D_2	Damage Fn Quadratic Coeff	/4.06E-5 /
C_2	Abatement Cost Fn Quadratic Coeff	/18 /
g	Stock Decay Rate	/0.005 /
I	Discount Rate	/0.01 /
α	Damage Nonautonomous Growth Rate	/0 /
β	Gross Emissions Nonautonomous Growth Rate	/0 /
θ	Abatement Cost Nonautonomous Growth Rate	/0 /
$\underline{D}_F$	Future Damages per Unit Pollutant Stock	/0.1/
Y	Time Step (Years per Discrete Time Step/Period)	/1/

VARIABLES {unrestricted in sign}

$B_E(t)$	"Emission Flow Permit Bank Account"
$B_S(t)$	"Emission Stock Permit Bank Account"
D_F	"Final (Terminal) Valuation/Damages of Durable Stock"
J	"Objective Function Value, NPV Costs ($ Bill)";

POSITIVE VARIABLES

A(t)	"Emission Abatement"
E(t)	"Emissions After Abatement"
S(t)	"Pollutant Stock"
D(t)	"Damage/losses Due to Pollutant Stock"
C(t)	"Abatement or Control Costs"

PARAMETERS

$E_U(t)$	"Gross Unabated Emissions (Bill Tons)"
δ(t)	"Discount Factor"

$\underline{\delta}$(t)	"Discount Factor with term for infinite stream in terminal period"
$\underline{\delta}$(T)	"Discount Factor for terminal period (T+1?) infinite stream"
MC(t)	"Marginal Abatement Cost ($/ton)"
MD(t)	"Marginal Damages ($/ton)"
MMC(t)	"Second Derivative of Abatement Cost fn"
APCT(t)	"Abatement as Percent of Emissions"
P_E(t)	"Socially Opt Emission Permit Price"
P_S(t)	"Socially Opt Stock Permit Price"
$\underline{E}$(t)	"Emission Permit Levels"
$\underline{S}$(t)	"Stock Permit Levels"
$\Delta\underline{S}$(t)	"Change in Stock Permit Levels"

PARAMETERS { for reporting across cases }

ROBJ(*)	"Objective Value"
RDISCFACTT(T,*)	"Discount Factor with Infinite Stream Terminal Period"
RDISCLAST(*)	"Discount Factor for T+1 Period Damages Taken Account of at time t"
RA(t,*)	"Emission Abatement"
RE(t,*)	"Emissions After Abatement"
RS(t,*)	"Pollutant Stock"
RMD(t,*)	"Marginal Damage Due to Pollutant Stock"
RMC(t,*)	"Marginal Abatement or Control Costs"
RBE(t,*)	"Bank Account For Emissions Flow Permits"
RBS(t,*)	"Bank Account For Emissions Stock Permits"
RMDMC(T,*)	"Difference Between MD and MC, MD-MC"
RSLAST(*)	"Stock in T+1"

Parameter Initializations

$E_U(t) = E_0\, e^{\beta tc}$	{unabated emissions grow at rate BETA}
$*\delta(t) = e^{-it}$	{contin-time disc factor for each period}
$\delta(t) = (1+i)^{-t}$	{discrete-time disc factor for each period}
$P_E(t) = 0$	{ initial emission permit price zero }
$P_S(t) = 0$	{ initial stock permit price zero }

EQUATIONS

EEQ(t) "EMISSIONS AFTER ABATEMENT EQUATION (Bill. Tons)"
Emissions E are unabated emissions EU less abatement A.

$$E(t) = E_U(t) - A(t)$$

DEQ(t) "DAMAGE DUE TO POLLUTANT STOCK ($ Bill)"

Quadratic stock pollutant damages.

$$D(t) = D_1 S(t) + D_2 S(t)^2$$

CEQ(t) "ABATEMENT OR CONTROL COSTS ($ Bill)"
Purely quadratic abatement costs.

$$C(t) = C_2 A(t)^2$$

FEQ(t) "RECURSIVE FINAL PERIOD STOCK VALUATION DEF"
Salvage value for accumulated capital.

$$FINALVAL(T) = \delta(T) S(T) F_V$$

SEQ(t) "POLLUTANT STOCK EVOLUTION (Bill. Tons)"
Stock evolution with decay, new emissions after abatement.

$$S(t+1) = S(t)(1-g) + E(t)$$

EBEQ(t) "EMISSIONS FLOW BOUNDS CONSTRAINT EQUATION"
Emissions flow limit equation.

$$E(t) \leq \underline{E}(t)$$

SBEQ(t) "EMISSIONS STOCK BOUNDS CONSTRAINT EQUATION"
Emissions stock limit equation.

$$S(t) \leq \underline{S}(t)$$

BEEQ(t) "BANK OF FLOW PERMITS EVOLUTION (Bill. Tons)"
Stock evolution with new permits less new emissions.

$$B_E(t+1) = B_E(t) + \underline{E}(t) - E(t)$$

BSPERMEQ(t) "BANK OF PERMANENT STOCK PERMITS EVOLUTION (Bill. Tons)"
Bank evolution with new permits, less new emissions, plus decay

$$B_S(t+1) = B_S(t) + \Delta\underline{S}(t) - (E(t) - S(t)g)$$

{ Emission Stock Bank Account Evolution - Temp Stock Permits }
BSTEMPEQ(t) "BANK OF TEMPORARY STOCK PERMITS EVOLUTION (Bill. Tons)"

Bank evolution with new permits, less new emissions, plus decay.

$$B_S(t+1) \;=\; B_S(t) + \underline{S}(t) - S(t)$$

BEFINALEQ(T) "FINAL BOUND ON BANK ACCT OF EMISSION FLOW PERMITS"

$$B_E(T) \;\geq\; 0$$

BSFINALEQ(T) "FINAL BOUND ON BANK ACCT OF EMISSION STOCK PERMITS"

$$B_S(T) \;\geq\; 0$$

SOBJEQ "SOCIAL OBJECTIVE FUNCTION DEFINITION"

SOBJEQ.. {NPV of Social damage and control costs}

This includes the sum over time of discounted quadratic stock pollutant damages and purely quadratic abatement costs, plus the Salvage value for accumulated terminal stock>

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t)\left(D_1 S(t) \;+\; D_2 S(t)^2 \;+\; C_2 A(T)^2\right) \;+\; \delta(T) S(T) F_V$$

POBJEQ "PRIVATE OBJECTIVE FUNCTION DEFINITION"

NPV of control costs and permit costs.

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t) C_2 A(t)^2$$

Sum over time and region of purely quadratic abatement costs.

PTAXOBJEQ "PRIVATE OBJECTIVE FUNCTION DEFINITION WITH TAXES"

NPV of control costs and permit costs.

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t)\left(P_S(t) S(t) \;+\; P_E(t) E(t) \;+\; C_2 A(T)^2\right) \;+\; \delta(T) S(T) F_V$$

This includes, for each period, payments for stock permits, payments for emission permits, and purely quadratic abatement costs. In the final year it also includes the Salvage value for accumulated capital.

Model Declarations

Model names are declared and then assigned equations by including equation names.

The social optimum model minimizes social damages plus control costs, where control costs depend on abatement effort, and damages _could_ depend on both emission flows and stocks. The constraints on cost minimization are:

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

```
MODEL SOCIALOPT   SOCIAL OPTIMUM STOCK POLLUTANT MODEL
    /
      EEQ, SEQ, SOBJEQ
    /;
```

The private optimum model minimizes private damages plus control costs, where control costs depend on abatement effort, and private damages are zero.
The constraints on cost minimization are:

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).
For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVOPT     PRIVATE DECISION UNCONSTRAINED
    /
      EEQ, SEQ, BEEQ, BSEQ, POBJEQ
    /;
```

The private permit model minimizes private damages plus control costs, given emission and stock permit levels and no banking/borrowing. Here control costs depend on abatement effort, and private damages are zero.
The constraints on cost minimization are
    emission flows and stocks bounded by permits in each period (EBEQ,SBEQ),
    the relation between emissions level and abatement level (EEQ), and
    the pollutant stock evolution equation (SEQ).
For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVPERM    PRIVATE DECISION GIVEN PERMITS AND NO BANKING
    /
      EBEQ, SBEQ,
      EEQ, SEQ, BEEQ, BSEQ, POBJEQ
    /;
```

The private banking model minimizes private damages plus control costs, given emission and stock permit levels and banking/borrowing. Here control costs depend on abatement effort, and private damages are zero.

```
MODEL PRIVBANK    PRIVATE DECISION GIVEN PERMITS AND BANKING
    /
```

```
    BEFINALEQ, BSFINALEQ, {only require final bank balances positive}
    EEQ, SEQ, BEEQ, BSEQ, POBJEQ
  /;
```

The private optimum model minimizes private damages plus control costs, plus the payments for emission or stock fees/taxes. Here control costs depend on abatement effort, and private damages are zero. The constraint on cost minimization are the relation between emissions level and abatement level (EEQ), and the pollutant stock evolution equation (SEQ). For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVTAX     PRIVATE DECISION UNCONSTRAINED
  /
    EEQ, SEQ, BEEQ, BSEQ, PTAXOBJEQ
  /;
```