Topics on Modeling New Technology Introduction: Learning-by-Doing, Irreversible Investment, Risk Aversion, and Limited Foresight

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I. Learning-by-Doing

Summary

"Learning-by-doing" is the effect of improving productivity (or reducing cost) by repetition of the production process. This phenomenon is observed in various industrial situations where it is described as a learning curve, progress function, or experience curve. It is important to distinguish learning-by-doing and learning by technological progress. (The latter is closely related to R&D spending.) Normally, pure learning-by-doing is the effect of productivity improvement simply by repetition of production process --- it's the "autonomous" accumulation of understanding and expertise. Thus, strictly speaking, the learning curve contains more information than learning-by-doing. On one hand, learning-by-doing, economies of scale and technological progress are different aspects of productivity improvement and should be discriminated one from another. On the other hand, since the production process is dynamic in nature and involves many factors, learning is an on-going and very complicated process. In the real world, it is hard or even impossible to clearly distinguish learning-by-doing from economies of scale and technological progress.

Arrow's classic form of the learning effect is represented by

$$A(G) = bG^{-n}$$

where G is cumulative gross investment (embodied in the existing capital equipment); A(G) is the labor cost; b and n are constants. The original simple model only considers labor cost reduction in the production process and assumes that at any time, new capital goods incorporate all the knowledge then available, but once built, their productive efficiency can't be altered by subsequent learning. The important question is which index of experience is the best. While cumulative output gives much stronger results than calendar time, cumulative output and cumulative investment have close results, although the latter is slightly better.

Lieberman(1984) is a typical empirical study of the learning curve. Lieberman collected data on 37 chemical products (including methanol) and uses a regression model to test the existence of learning effect. The basic model is similar to Arrow's classic model, excepting Lieberman uses cumulative output and cumulative investment and even their weighted combinations as the indexes of experience. The results exhibit a strong and consistent learning effect. Learning is found to be a function of cumulative production and cumulative investment rather than the calendar time. Standard economies of scale appear significant but small in magnitude relative to the learning effect. Variations in the slope of the learning curve are linked to difference in R&D spending and capital intensity.

Uncertainty makes the learning less important when determining current production levels. The reason is that a learning curve affects production by making part of the firm's production costs equivalent to an irreversible investment in reduced future costs. Uncertainty over future prices creates an opportunity cost that reduces the net benefit from the investment.

Table
Empirical Evidence on Rates of Learning-by-Doing

Study	Data or Method	Results
Sheshinski (1967)	22 regressions, cross-nation, cross-industry data Ln(A)=a+b ln(G), A=productivity G=Cumulative experience a,b=coefficients	b ranges from 0.7 - 0.85
Baloff (1971)	regression on 3 labor-intensive factories (start-up model)	doubling cumulative output, unit costs reduced by: 10%-12.5% musical instruments 20%-25% in apparel production 10%-20% in auto assembly
Hart (1983)	1929-1968 U.S. primary magnesium production	1% cumulative output increase, 0.16% price decrease
Lieberman (1984)	37 products in U.S. chemical processing industries (includes methanol)	Unit cost decreases on average 5.5% a year, economies of scale account for 15% of decrease, while cumulative output increases accounts for 85%
Adler and Clark (1991)	2 manufacturing dept. of an electric company	second-order learning accounts for about 70% of the total learning effect

"Learning by doing" is the effect of improving productivity (or reducing cost) by repetition of the production process. This phenomenon is observed in various industrial situations where it is described as a learning curve, progress function, or experience curve.

The study of learning curve was initiated by Wright(1936). Early work also includes Hirsch(1952), Arrow(1962) and Rapping(1965). From late 60s to mid-70s, the Boston Consulting Group (BCG) made the "experience curve" a focus of its research. During late 70s and early 80s, some important articles were published, including some empirical studies. Recent years, research on learning curve was centered on the market issues, mathematical modeling and algorithm and other issues related to learning, such as externality, investment, uncertainty, behavioral implications. Many studies also featured empirical investigation of learning effect in various industries. Despite all these efforts, later models of learning curve still follow the basic structures of the early models, and the evidence on the precise nature of the learning is still quite limited and sometimes even mixed.

Learning curve study normally covers the fields of economics, industrial engineering, operations research, management science as well as cost engineering. This summary tries to outline the key results of the major studies on this topic.

1. Arrow's Classic Model and Extensions

Arrow(1962) argues:

a. Learning is the product of experience.

b. Learning by repetition of a problem is subject to diminishing returns. Thus, to have continuous learning one needs continually to meet new problems. Arrow considers cumulative output as an index of experience but rejects it on the grounds that, if the rate of output is constant, then the stimulus to learning would seem to be constant. He thus takes cumulative gross investment as an index of experience. The learning effect in his model is essentially represented by

$$A(G)=bG^{-n}$$

where G is cumulative gross investment. (embodied in the existing capital equipment). A(G) is the labor cost. b and -n are constants. There are two main drawbacks of Arrow's model: First, it assumes that at any time, the new capital goods incorporate all the knowledge then available, but once built, their productive efficiency can't be altered by subsequent learning. second, it only considers labor cost reduction in the production process.

The interesting question is which index of experience is the best. Arrow uses cumulative gross investment. Some authors use cumulative output. Arrow's argument may have some justification, but it isn't so convincing. Later studies by Rapping(1965) and Sheshinski(1967) show that while cumulative output gives much stronger results than calendar time, cumulative output and cumulative investment have close results although the latter is slightly better. In later studies, various indexes of experience were used, but the most frequently used was the cumulative output.

Baloff(1971) extended Arrow’s basic model by a "start-up" model. He argued that the cost reduction process included two phases, the start-up phase and the steady-state phase. During start-up phase, the cost curve is downward sloping. The cost is constant during steady-state phase. The slope of cost curve, the rate of learning effect on cost reduction, depends on various industrial factors.

2. BCG’s Study

BCG did a cross-industry (also across-country) empirical study in 1972. It examined the learning effect of 24 products in U.S., G.B. and Japan. The model has the same equation as Arrow’s, excepting for the G, which stands for cumulative output here. If there exists learning effect, the learning curve is expected to be downward sloping. The results: of the 24 products, 6 are not consistent with the above model, while other 5 exhibit upward sloping curves. (these products include crude oil and motor gasoline) BCG claims that experience curve apply essentially to specific products rather than to companies. BCG also finds that the rate of cost reduction is usually slow at first and then followed by a much faster decline. (kinked cost curve) If this conclusion is true, it may serve as the extension of Baloff’s start-up phase study.

Hart(1983) commented on the British government’s Green Paper of 1978. Hart points out that successful firms are lower down the learning curve than are other firms, and their lower costs and prices increase market share and profitability. This argument and BCG’s study raise the question as whether learning is appropriable by firms or by industry. Evidence seems again mixed. While products may have "built-in" learning effect as cumulative output increases, the slope of the learning curve still heavily depends on production environment of specific firm. Otherwise, firms will not have incentive to be first. (Being first normally means high risk.) The factors contribute to the specific firm environment may include the labor quality, the existing equipments, the firm’s attitude to new technologies and the firm’s management system of production. Thus under normal conditions, learning should be regarded as appropriable at product, firm and industry levels. Most products may exhibit learning effect of the combinations of the three levels. Different products may have different weights of the three learning levels. These conditions are also closely related to technology diffusion, the patent system and many other factors.

3. Learning By Doing and Market Issues

Spence(1981) is an important paper. It is primarily concerned with learning as an entry barrier. Spence argues that the firm should produce to the point where current marginal cost exceeds revenue, because an incremental unit of current production reduces future production costs by moving current the firm down the learning curve that partly offset its cost. Chu(1988) finds that when entries do not take place at the same time, speed of learning primarily affects how effectively latecomers can compete with an early entrant.

Fudenberg and Tirole(1983) uses a general continuous-time model to show that output increases over time in the absence of strategic interactions. It also uses a two-period model to analyze the case in which firms do consider the effect of their learning on the actions of their rivals. It shows that Strategic incentives can induce firms to choose decreasing output paths, and that a little diffusion of learning across firms increases output if firms are naive, but decreases output if firms play strategically. In the absence of strategic interactions, a monopolist learns too slowly when compared with social optimum. If there is no diffusion of learning and there exist strategic interactions , firms will have incentive to choose decreasing output paths to gain more advantage of being first. The diffusion of learning will lower the learning curve of the "naive" firms(free-riding), and these firms may choose increasing output paths.

Devinney(1987) shows that the patterns of price and cost movements of the existing firms will be greatly affected by the entry of new competitors. If firms have U-shaped average cost curves, entry alone can cause price and cost declines that look like learning-induced reductions. Alternatively, cost increase may occur even when learning is quite strong. This suggests that the learning process is quite complicated and we should be aware of the role that entry plays in affecting cost.

4. Lieberman’s Study and Bahk and Gort’s Decomposition of Learning

Lieberman(1984) is a typical empirical study of learning curve. Lieberman collects data of 37 chemical products (including methanol) and uses regression model to test the existence of learning effect. The basic model is similar to Arrow’s classic model, excepting Lieberman uses cumulative output and cumulative investment and even their weighted combinations as the indexes of experience. The result exhibits a strong and consistent learning effect. Learning is found to be a function of cumulative production and cumulative investment rather than the calendar time. Standard economies of scale appear significant but small in magnitude relative to the learning effect. Variations in the slope of the learning curve are linked to difference in R&D spending and capital intensity.

Lieberman also raises the question of distinguishing learning by doing and learning by technological progress. (The latter is closely related to R&D spending.) Normally, pure learning-by-doing is the effect of productivity improvement simply by repetition of production process --- it’s the "autonomous" accumulation of understanding and expertise. Thus strictly speaking, learning curve contains more information than learning by doing. On one side, learning by doing, economies of scale and technological progress are different aspects of productivity improvement and should be discriminated one from another. On the other side, since the production process is dynamic in nature and involves many factors, learning is an on-going and very complicated process. In the real world, it is hard or even impossible to clearly distinguish learning by doing from economies of scale and technological progress.

Bahk and Gort(1993) provides a possible decomposition of learning by doing. It a the learning by doing in the context of a production function in which the other arguments are labor, human capital, physical capital, and vintage as a proxy for embodied technical change in physical capital. Learning is further decomposed into organization learning, capital learning, and manual task learning. The model is tested with time-series and cross-section data for various samples of up to 2,150 plants over a 14-year period.

Globerson and Levin(1987) argues that we should incorporate both learning and forgetting into organizational environments. Variables that affect forgetting level are: 1. turnover rate, 2. communication level, 3. intensity of documentation, and 4. intervention time between repetitions.

5. Learning By Doing and Uncertainty

Majd and Pindyck(1989) examines the implications of the learning curve in a world of uncertainty. It considers a competitive firm whose costs decline with cumulative output. Because the price of the firm's output evolves stochastically, future production and cumulative output are unknown and are contingent on future prices and costs. An optimal decision rule that maximizes the firm's market value is: produce when the price exceeds a critical level, which is a declining function of cumulative output.

Uncertainty seems to make the learning less important when determining current production levels. The reason is that a learning curve affects production by making part of what the firm spends to produce an irreversible investment in reduced future costs. Uncertainty over future prices creates an opportunity cost that reduces the net benefit from the investment. Majd and Pindyck(1989) also shows how the shadow value of cumulative production, the total value of the firm, and the decision to produce depend on the volatility of the price and other parameters. Uncertainty increases the critical price required for the firm to produce, but also increases the value of the firm. Thus, during periods of high volatility, firms facing a learning curve ought to be producing less, but are worth more.

6. Adler and Clark's Learning Process Model

As noted earlier, the precise nature of learning process is very complicated. There are too many factors involved in the process. Adler and Clark(1991) attempts to investigate the nature of the learning process. Using data from two manufacturing departments in an electronic equipment company, Adler and Clark construct a model of productivity improvement as a function of cumulative output and two managerial variables --- engineering changes and work force training. Each of the two variables is linked to the more traditional source of learning --- the learning by doing.

It uses two learning models: 1. the classic learning curve model, in which productivity is an exponential function of experience, and 2. the learning process model, which is premised on the idea that a significant part of the effect of experience on productivity captured in the learning curve model might be due to the influence of identifiable managerial actions. The standard learning curve model is compared with the learning process models of the two departments. Three main conclusions are generated: 1. The total learning effect is just as strong in the capital-intensive area as in the labor-intensive and materials-intensive area. 2. The learning process is internally complex. 3. The relative roles of explicitly managed, 2nd-order learning and of tacit, first-order learning vary substantially across processes.

Above is a summary of major literature regarding learning effect in production. As I pointed out earlier, the knowledge of the nature of the learning process is still quite limited and sometimes even mixed. This does not suggest that learning effect is not important for decision-making at the firm, industry levels, but rather, shows that more creative research should be done on this particular subject.

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II. Investment in Fuel and Vehicle Production Capacity

Stylized Facts

1) Short-run production of alternative fuels and vehicles is limited by production capacity.
2) Future demand for alternative fuels and vehicles is uncertain because alternative fuels and vehicles are complementary products whose individual demand is uncertain and whose complementary availability is not assured.

Question: When will firms invest in future capacity?
Question: What happens to short-run profits with and in the absence of new capacity?.
Question: How does demand uncertainty translate into price uncertainty and profits?

Traditional Investment Theory

The standard theory of investment behavior compares the net present value of a marginal investment to its purchase price. This theory has developed along two lines but comes to the essentially the same conclusions. The neoclassical approach (Jorgenson (1963), Gould (1968), Arrow (1968)) compares the marginal value of an incremental unit of capital with an equivalent per period rental cost of capital computed from the purchase price and other factors. The firm's desired stock of capital is found by equating the marginal product and the user costs. The actual stock is assumed to adjust to the ideal, through some assumed lagged process or as a response to an assumed adjustment cost mechanism. A second approach stems from Tobin's(1969) q-theory of investment. In this approach, Tobin compares the capitalized value of a marginal investment to its purchase cost. This stems from the insight that if an additional unit of capital contributes more to present discounted profits than its purchase price plus installation costs, then a profit maximizing firm will purchase the additional unit of capital. For private firms, the value of a marginal unit of investment is equal to the stream of profits expected from the investment.

The neoclassical theory of investment rests upon the behavioral assumption that firms maximize the present value of net cash flows subject to constraints on production and capital accumulation. As shown formally below, net worth is defined as the integral of discounted net revenues. All prices including the interest rate are assumed fixed. More generally, this theory assumes that the expected values of prices and the interest rate are fixed or known. The central feature of neoclassical theory is that investment demand for capital responds to changes in relative factor prices or the ratio of factor prices to the price of output. Firms choose the amount of variable inputs (labor) to hire, as well at the rate of investment. In addition, a net change in the capital stock is usually assumed to impose costs on firms that rise with the rate of investment. These adjustment costs are justified on the ground that if the capital stock is to be increased by a given increment, it is more costly to achieve this increase rapidly rather than slowly. In a sense, neoclassical investment theory should really be called dynamic theory of the firm.

Able (1990) provides a formal consistent treatment of firm investment theory showing the links between the neoclassical theory and that of Tobin. Able's notation is used below. Denoting $V_t^*(\beta)$ be the value of a firm at time t, the value of the firm is calculated as:

$$V^*(\beta)_t = \underset{I_t L_t}{Max} \int_t^{\infty} e^{-rt}\left[Y(Kt,Lt) - w_t L_t - p_t I_t - C(I_t,K_t)\right] \quad \textbf{(3)}$$

$$s.t: \qquad \dot{K}_t = I_t - hK_t \quad \textbf{(4)}$$

where:
$Y(K_t,L_t)$ is the firm's revenue at time t;
I_t , K_t and L_t are investment, capital and labor, respectively, at time t;
$C(I_t,K_t)$ are adjustment costs at time t;
w_t and p_t re the prices of labor and capital at time t, while h is the rate of capital depreciation; and, the dot over K denotes the derivative of the variable for capital stock with respect to time, i.e. the dot indicates the instantaneous rate of change in the capital stock.

At each moment in time the firm chooses the quantities of labor and investment to maximize the net cash flow (1) subject to the capital accumulation equation (2). To solve this problem one maximizes the current value Hamiltonian defined as:

$$H_t = Y(Kt,Lt) - w_t L_t - p_t I_t - C(I_t,K_t) + q_t \dot{K}_t \quad \textbf{(5)}$$

$$q_t \dot{K}_t$$

In (3) q_t can be interpreted as the shadow value of a unit of installed capital. Thus, a firm's labor and investment decisions produce a cash flow equal to revenue minus expenses and increase or decreases in the value of the capital stock by an amount.

Solving this problem leads to two conditions for hiring labor and investment:

$$Y_L(K_t,L_t) = w_t \quad \textbf{(7)}$$

$$c_I(I_t, K_t) = q_t - p_t. \qquad \textbf{(8)}$$

Equation (5) can be interpreted as requiring firms to hire labor up to the point at which the marginal revenue product of labor equals the price of labor (the wage rate). Equation (6) can be interpreted to require that wealth maximizing firms choose a rate of investment such that the marginal adjustment costs plus the per unit costs of investment equal the value of an additional unit of installed capital.

A standard specification of investment assumes that there is an exogenous mechanism that determines the rate at which the gap between the desired capital stock at some time t (K_t^*) and the actual capital stock is closed. A general form of Jorgenson's investment equation making this assumption is:

$$I_t = \left\{ \sum_{i=0}^{n} \omega_i [K_{t-i}^* - K_{t-i-1}^*] \right\} + hK_t . \qquad \textbf{(9)}$$

Other assumptions are made and yield somewhat different specifications. In the accelerator model, investment is a distributed lag function of changes in the level of firm revenue:

$$I_t = \left\{ \sum_{i=0}^{n} \omega_i [Y_{t-i} - Y_{t-i-1}] \right\} + hK_t . \qquad \textbf{(10)}$$

This model can be seen as a special case of the neoclassical investment model by Jorgenson when the user cost of capital is ignored or when the elasticity of substitution between capital and labor is zero.

Tobin's q theory of investment (1969) postulates that the incentive to build new capital depends on the market value of the capital relative to the cost of constructing the capital. If an additional unit of installed capital would raise the market value of the firm by more than the cost of purchasing the capital plus adjustment costs, then firms will acquire it. Tobin defined the variable q to be the ratio of the market value of a firm to the replacement cost of its capital.

$$q_t = \frac{V_t}{(p_t K_t)} \qquad \textbf{(11)}$$

Given certain conditions, including that a firm's cash flow is a linearly homogeneous function of K, L and I, Tobin's q theory of investment is consistent with the adjustment cost theory of investment (see Hayashi, (1982) and Able (1990) for details). Tobin's q is popular because the stock and bond markets can be used to value firms' capital stocks. There are, however, econometric difficulties from investment equations based on Tobin's q. These equations typically have large unexplained serially correlated residuals (Able (1990)).

The theoretical underpinnings of investment demand, however, do not lead to specific investment demand equations for which explains the demand for observable variable for a given industry or firm. Comparisons of different econometric specifications of investment demand models are given by Jorgenson, Hunter and Nadiri (1970a 1970b), Nickell (1978), Able (1990), and the references therein.

Investment Under Uncertainty When Investment is Partly or Completely Irreversible

The conventional wisdom described above was based on the assumption that investment is reversible or that the investment is irreversible but that it is a now or never proposition. However, investment decisions concerning the production of alternative fuels and vehicles generally do not fall under that classification. This is because the decision to invest in a methanol plant, for instance, does entail considerable future demand (price) uncertainty and sunk costs, and in addition, is not a now or never proposition. Investors are perfectly free to delay investment decision to acquire more information about the desirability or timing of investing. Sunk costs arise on the firm and industry level since purchased capital cannot generally be used outside the industry, or cannot be used as profitably outside the intended industry. Recent research shows that there exists an opportunity cost to investing today; the opportunity cost is the value of an option to invest (see Dixit and Pindyck (1994), Pindyck (1991) for excellent surveys of the literature).

This line of research usually posits that the value, V, of an investment is uncertain and follows geometric Brownian motion of the form:

$$dV = \alpha V dt + \sigma V dz \qquad \textbf{(12)}$$

where dz is an increment of a wiener process, α and σ are trend and variance parameters respectively. This equation implies that the current value of a project is known, but that the future values are lognormally distributed with a variance that grows linearly with time. In this context, the investment rule is now invest when $V \geq V^*$, where V^* is a critical value which depends in part on α and σ. For reasonable parameter values, McDonald and Siegel (1986) show that it is optimal to defer investing until the present value of benefits from a project are twice as large as the investment costs. Moreover, increases in uncertainty increase the return needed to invest in a project. Thus, the traditional approach described above could be substantially in error when investment decisions are predicated on uncertain demand and irreversible investments.

Very little empirical work has incorporated the option to invest into econometric or simulation models. This is because the techniques are relatively new and require numerical solutions to second-order nonlinear partial differential equations. The necessary parameters include risk-adjusted rates of return, annual price trends and price trend variances, and specific costs and lifetimes of different technology options. Nonetheless, several empirical applications have be completed. A review of empirical applications appears in Dixit and Pindyck (1994).

Is there market failure in private firm investment behavior?
Does uncertainty in the future demand for fuels and vehicles justify government intervention? Does demand (price) uncertainty cause private firms to make decisions that are non-optimal from a public perspective? Not in general, only if firms face a different value of waiting than does society as a whole. If markets for risk are incomplete, then government intervention is justified.

Policy Conclusions and Future Research

It is feasible to use the conventional neoclassical theory described above to estimate investment demand equations for produces of alternative fuels and vehicles. For fuel producers (methanol, ethanol & perhaps LPG, and LNG) this would be fairly straightforward since these fuels are currently produced commercially (though not at the same scale as would occur with significant new transportation demand.) It would also be feasible to estimate investment demand equations for alternative fuel vehicles, but since these vehicles are not now produced commercially, the equations would have to be estimated based upon data from gasoline vehicles. *Given known future demand* for alternative fuels and vehicles these equations could be expected to provide information about timing and returns to producers.

The accuracy of investment demand equations based on traditional theory is limited by the magnitude of uncertainty about future demand. Traditional theory usually incorporates a risk premium into the cost of capital to account for this uncertainty. The risk premium itself must, of course, be estimated.

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III. Expectations in Dynamic Modeling

1. Static (Myopic) Expectations

Static expectations (myopia) assumes that a variable in the future will be:
A) of the same value as in the present.
B) will grow or decline in value at the same rate as the present.

$$\text{A)}\quad E_t x_{t+1} = x_t, \text{ or}$$
$$\text{B)}\quad E_t x_{t+1} = (1 + r)^t x_t$$

In model (B), the parameter r is estimated based on historical data.

In an extreme case, agents (firms, consumers) would form expectations in some initial period and not update them over the entire planning horizon.

2. Adaptive Expectations

Expected future conditions are linear combination of lagged historical conditions.

$$E_t x_{t+1} = \sum_{i=0}^{\infty} \alpha_i x_{t-i}$$

In this model, the α_i parameters are estimated based on historical data. There are many variations of the distributed lag formulation which incorporate various types of restrictions (assumptions). For example the following form is restrictive in that it assumes that the most recent observations are those that are most important; observations in the past decline in importance at a geometric rate.

$$E_t x_{t+1} = \sum_{i=0}^{\infty} \lambda(1 - \lambda)^i x_{t-i} \quad 0 < \lambda < 1$$

The distinction between adaptive and myopic expectation is that myopic expectations are a special form of adaptive expectations where all lagged term coefficients are zero except for that applying to the current term.

3. Rational Expectations

Current expectations reflect accurate knowledge of future mean. Future outcomes are, on average, consistent with current expectations. Current expectations are an unbiased estimate of future conditions.

$$E_t[x(t+1)] = E_{t+1}[x(t+1)] = \bar{x}(t+1)$$

4. Perfect Foresight

Agents make decisions based on accurate knowledge of current and future conditions (actual outcomes, not just expectations.

$$E_t[x(t+1)] = x(t+1)$$

Solution Algorithms and Foresight Assumptions

1. Simultaneous solution of a dynamic model is equivalent to a perfect foresight assumption.
2. Period-by-period solution is equivalent to static expectations