Efficient Greenhouse Gas Emission Banking and Borrowing Systems

by

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Abstract

There is tremendous international interest in controlling emissions of greenhouse gases. One of the most prominent proposals, called 'joint implementation', relies on the international trading, of global warming permits or credits. The U.S. government has suggested that this trading regime also allow nations the additional flexibility provided from the banking and borrowing emissions over time. Intertemporal emission permit trading systems that allow banking and borrowing cause permits to be arbitraged across time according to the present value price of permits. To date, international attention has rightly focused on the setting the initial endowments of permits and on the rules to insure that promised reductions do in fact occur. Overlooked, however, is how one ought to set the rules for the intertemporal permit banking and borrowing. If the regulators specify an intertemporal trading rate for banking and borrowing, then they determine the time rate of change in permit prices. The socially optimal banking system for a stock pollutant (i.e., a pollutant whose damages depend on its accumulated stock) such as greenhouse gases will depend upon the efficient growth rate of marginal stock damages. In particular, the optimal growth rate of permit prices, and therefore the optimal intertemporal trading rate, is equal to the ratio of current marginal stock damages to the discounted future value of marginal stock damages less the decay rate of emissions in the atmosphere. When there is a difference between private and public (individual and collective) discount rates, the flow-permit banking interest rate must be increased by that difference. To numerically estimate the "interest" rate that should be offered on greenhouse gases permit bank accounts, we use values from the literature and perform experiments with publicly available global climate-economic models. Sensitivity analysis indicates how confident the parties to a climate change treaty might be that a particular banking and borrowing regime will actually improve global welfare.

1.0 Introduction

Following the signing of the Framework Convention on Climate Change at the 1992 United Nations Conference for Environment and Development in Rio, which calls for the stabilization of greenhouse gas concentrations in the atmosphere at 1990 levels, a growing number of researchers and policy makers have proposed permit trading in greenhouse gasses (GHGs) (e.g., Falk and Mendelsohn (1993), Hahn and Stavins (1993), Swart (1993), Kosobud et al. (1994), Jackson (1995)). While appropriately recognizing the stock nature of the problem, none of this research has investigated the properties of intertemporal GHG permit trading in a general framework that allows the flexibility afforded when permits may be traded, banked and, possibly, borrowed.

At the same time, however, recent work has begun to investigate the properties of intertemporal permit systems for *flow* pollutants; pollutants whose deleterious effects are solely a function of the current flow rate (Rubin and King (1993), Biglaiser et al. (1995), Cronshaw and Kruse (1996), Rubin (1996), Kling and Rubin (1997)). In examining flow pollutants, these papers use intertemporal models which allow firms to bank (Cronshaw and Kruse (1996)) and bank and borrow (Rubin and Kling (1993), Rubin (1996)) emissions through time in addition to the inter-firm trading which characterizes single-period permit systems. Kling and Rubin (1997) show that unrestricted emission banking and borrowing of flow pollutants is not necessarily socially optimal.[1] This arises because unrestricted permit banking and borrowing causes discounted permit prices and, therefore, discounted marginal abatement costs to be equalized through intertemporal arbitrage by private agents. At the same time, however, there is no reason to presume that the resulting emissions path is socially optimal, since the social optimum requires, for stationary damage and costs functions, that current value marginal abatement costs should be constant across time.[2] However, as Kling and Rubin (1997) show, the banning of flow permit banking and borrowing is also not optimal.

In the case of a *stock* pollutant, that is, one where damages depend upon the accumulated stock, permit banking is even more problematic. For stock pollutants, there is no reason to believe that marginal damages are equal in different periods. Indeed, the behavior of individual agents (firms or nations) can well diverge from the social optimum when intertemporally trading stock pollutants. The issue is how to devise an efficient banking regime.

Permit systems that allow banking and borrowing (hereafter bankable permits) are seeing growing regulatory interest both nationally and internationally. The sulfur dioxide trading program, authorized by the 1990 Clean Air Act Amendments, is the best known and most extensive venture

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[1]Biglaiser et al. also show that an intertemporal permit trading will not be optimal. The model used by Biglaiser et al., however, does not allow for the borrowing and banking of permits, but rather looks at trading lifetime rights to emit flow pollutants.

[2]We will consider the cases of non-stationary damages and control costs as well.

in marketable permits to date.[3] This program allows firms to bank, but not borrow, permits. Another domestic example is current fuel economy regulations that allow automobile manufacturers to bank *and borrow* fuel economy credits for up to three years (49 USC 32903). Certainly, however, the grandest use yet envisioned for marketable permits was contained in a recent draft proposal by the U.S. Department of State which would have allowed nations of the world to trade, bank and borrow greenhouse gas permits under the Framework Convention on Climate Change (USDOS, 1997).

Despite the reluctance of developing nations to allow any form of emission trading, the Kyoto Protocol signed last year does allow emission trading among Annex B (developed) nations (United Nation, Article 16bis 1997). The details on emission trading are to be negotiated in the future. Whether or not the banking and borrowing, in particular, of greenhouse gas permits will be allowed is not yet determined. The Kyoto Protocol essentially allows interest-free banking and borrowing within the first 5-year commitment period. Matters are less set for the second and future 5-year commitment periods. A carefully structured trading system that makes some provision for permit banking and borrowing has significant merit. It can maintain market incentives for efficient emission reductions while still providing individual parties the time-flexibility that they may need to meet their negotiated obligations. Furthermore, even if widespread intertemporal trading is not accepted in the near term, a better understanding of the marginal costs and benefits of moving emissions through time could also serve as a starting point for negotiating reasonable restitution (in dollars or in GHG-tons) by parties who are unable to meet their originally negotiated emission reduction schedule.

To date, international attention has rightly focused on the setting the initial endowments of permits and on the rules to insure that promised reductions do in fact occur. Overlooked, however, is what should happen if reduction goals are not met and how one might set the rules for the intertemporal permit banking and borrowing. A recent paper by Leiby and Rubin (1998) develops and solves a generalized intertemporal permit system for emissions that both cause damage instantaneously (i.e., “flow” damages), and also cause damages based on their accumulated stock (i.e., “stock” damages). Examples of this type of pollutant include the criteria pollutants (carbon monoxide, nitrogen oxides, and nonmethane volatile organic compounds) that can cause acute health affects *and* can promote the atmospheric concentrations of greenhouse gases, including carbon dioxide, methane, and ozone (EIA, pp. xiv, 61, 1995).[4] A permit system for the special case of emissions that only cause stock damages is also examined. The latter, simpler case corresponds roughly to the greenhouse gas emission reduction regime proposed by the U.S. Department of State.

Leiby and Rubin (1997) show that a bankable stock permit system can achieve the socially

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[3]See Burtraw (1994) and USGAO (1994) for overviews of the sulfur dioxide trading program.

[4]The magnitude of the global warming potential of criteria pollutants, however, depends on local atmospheric conditions (EIA, p. 61, 19995).

optimal pattern of emissions *provided* an efficient set of banking rules is devised. In particular, one efficient banking system would establish an intertemporal permit exchange rate by allowing banked permits to accumulate "interest," and by charging the same interest rate for permit borrowing. The efficient interest rate depends upon the time-rate of change of marginal damages along the socially optimal emissions path.

This paper investigates the empirical properties of a generalized intertemporal permit system for emissions that accumulate in the environment, where damages depend on the accumulated stock, such as GHGs. Damages from GHGs result from the warming of the earth's atmosphere caused by increased concentrations of the GHGs in the atmosphere. Different gasses have different instantaneous thermal effects and different total global warming potentials, where total global warming potentials account for the long-term warming that occurs over the life-time of the gases in the atmosphere (Lashof and Ahuja, 1995).

In particular, we use numerical models to estimate an optimal intertemporal trading ratio for the special case of a pure stock damage permit system. The optimal emissions trading rates depend on marginal abatement cost, marginal stock damages, and the decay rate of emissions. They also depend on whether permit allocations are viewed as temporary or permanent rights. These optimal permit systems are then compared with an alternative regulatory regime using pollution taxes and emission standards. Given some plausible parameter estimates taken from the literature, we conclude with policy recommendations.

2.0 Stock Pollution Permits

Mathematical Model of a Flexible GHG Banking System

Unlike flow pollutants, stock pollutants accumulate in the environment because their rates of emission into the environment exceed the environment's assimilative capacity. With flow pollutants, damages are solely a function of their instantaneous emission flows (rates). Damages from stock pollutants are a function, at each instant in time, of the level of accumulated pollution and possibly the contemporaneous flow. Letting S(t) be the total stock of all firms' emissions at any point in time, $S(t) = \sum_{i=1}^{N} S_i(t)$, we see below in (1) that whenever the sum of all firms' emission is greater than the natural decay of emissions, $\gamma S(t)$, then the stock of emission will be increasing. Here, emissions are taken to decay at a constant rate γ.[5]

$$\frac{dS}{dt} = \dot{S}(t) = \sum_{i=1}^{N} e_i(t) - \gamma S(t) \tag{1}$$

[5]In general, of course, the rate of decay need not be constant. For greenhouse gases it will vary with the gas of interest, and for some gases (e.g. CO_2) it may depend upon gas concentrations (stock). This simplification does not substantially affect our analysis.

In principle, an agency (national or international) regulating a stock pollutant might only be concerned with the time-integral of emissions (e.g. equation (2)) being less than a given standard at some point in time T. We call this type of standard a terminal stock standard.

$$S(T) \;=\; \int_0^T \dot{S}(t)dt \;+\; S(0) \;\;=\; \int_0^T (e(t) \;-\; \gamma S(t))dt \;+\; S(0) \;\le\; \bar{S}(T). \tag{2}$$

This type of standard is appropriate given threshold stock effects, i.e., a particular level of pollution cannot be exceeded without great damage. Implicit in this framework is that the rate at which the emissions accumulate is unimportant so long as the total allowable stock is not exceeded. The terminal stock standard could be generalized to a continuous or annual stock standard $\bar{S}(\tau)$ for each τ.

$$S(\tau) \;=\; \int_0^\tau \big(e(t) \;-\; \gamma S(t)\big)\, dt \;+\; S(0) \;\le\; \bar{S}(\tau) \quad \forall\; \tau \tag{3}$$

Alternatively, as suggested by Kosobud et al. (1994), a regulatory agency could set a series of emission rates, $\bar{e}(t)$. If the emission rates were constant through time this would assure (in the context of global warming) that developed nations would freeze their rates of emission of greenhouse gasses.

Permits are permanent if, once purchased, they provide a durable right over the T period horizon to current and future emission flows or stocks. Temporary permits, once purchased, provide a one-period right to a unit flow or stock (this right is instantaneous in the case of a continuous-time analysis). Regardless of the type of emission standard, be it based on a stock or a flow, permanent or temporary, the regulatory agency may allow firms or nations to bank and borrow permits. The negotiated limits for GHG emissions (Kyoto, COP3) best correspond to temporary flow permits for a stock pollutant. As of this writing, the specific regime under which trading is to occur remains to be negotiated.

Flow Permit Banking

With temporary flow permits, a firm or nation can be thought of as having a permit bank account B_{ei} which grows whenever its allocated (temporary) emissions permits $\bar{e}_i(t)$ plus any purchased flow permits $x_i(t)$ exceed its actual emission flow level $e_i(t)$.

$$\begin{aligned} B_{e_i}(t) \;&=\; \int_0^t (\bar{e}_i(\tau) \;-\; e_i(\tau) \;+\; x_i(\tau))d\tau \;+\; B_{e_i}(0) \\ \dot{B}_{e_i}(t) \;&=\; \bar{e}_i(t) \;-\; e_i(t) \;+\; x_i(t) \end{aligned} \tag{4}$$

The emission flows e_i are measured in tons per year and stocks S_i are measured in tons. Accordingly, the temporary flow bank account is measured in tons. The flow permit bank accounts B_e are each subject to a terminal non-negativity constraint to ensure that firms do not simply borrow or sell emissions which they never repay.

3.0 Intertemporal Emission Allocation From the National or International Perspective

In the national or international problem, the environmental regulator's objective is to maximize consumer and producer surplus less social damages from the good, $y(t) = \sum_{i=1}^{N} y_i(t)$,whose production causes instantaneous emission flows, $e(t) = \sum_{i=1}^{N} e_i(t)$, and cumulative emission stock $S(t)$.[6] Emissions are assumed to harm world or national welfare as described by the convex damage function $D(e(t), S(t),t)$, where $D_e(e,S,t) > 0$, $D_{ee}(e,S,t) > 0$, $D_S(e,S,t) > 0$, $D_{SS}(e,S,t) > 0$, and $D_{eS}(e,S,t) > 0$.

In the context of global warming each "firm's" emissions and output can be interpreted as each "nation's" emissions and output. The coordinating authority is not a single nation's government, but the UN member states acting collectively. Firm *i*'s minimum total cost of producing output $y_i(t)$ and unconstrained emission level $e_i(t)$ is $C_i(y_i(t),e_i(t),t)$. It is assumed that $C_i(y_i(t),e_i(t),t)$ is strongly convex in $(y(t),e(t))$ and with $C_y > 0$, $C_e < 0$, $C_{yy} > 0$ and $C_{ye} < 0$.[7] Therefore, higher levels of emissions are associated with lower production costs both total and at the margin. Given this notation, marginal abatement costs are denoted as $-C_e > 0$.

In modeling the optimal control of GHG emissions, or any other stock pollutant, one can choose to use an infinite time horizon and concentrate on the steady-state optimum conditions. Alternatively one can choose to use a finite time horizon (this could be very long indeed) and concentrate on the path of emissions and costs and damages. We agree with Falk and Mendelsohn (1993) that realistic time-dependent stock pollution problems do not define a steady state, and applying steady-state regulations to a dynamic path will necessarily be inefficient. Nonetheless, with stock pollutants, it is important to consider the damages that will occur from the built-up stock of pollution even after the finite regulatory program (and the finite analysis period) has formally ended. In both formal mathematical terms and empirically, taking consideration of terminal stocks is important and must be dealt with directly.

From a multi-year (continuous time, finite horizon) perspective, the welfare maximization problem

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[6]We use lower case symbols to denote flows, and upper case symbols to denote stocks. Symbols subscripted by ***i*** indicate variables for individual firms or nations, otherwise the symbols refer to national or global market totals.

[7]Here, subscripts which are variable names refer to the partial derivative with respect to that variable. In addition, the "i" subscripts indicating the firm under consideration and the functional dependency of variables on t will frequently be suppressed to reduce clutter.

is given below where the final value term $F(S(T))$ captures the value of damages for all time periods after T (measured in period T dollars) In addition to the terms already defined, P_y is the inverse demand curve for good y, and $\delta(t) = e^{-\rho t}$ is the instantaneous social (or collective) discount factor.

$$J^* \equiv \max_{\substack{e_1 \ldots e_N \\ y_1 \ldots y_N}} \int_0^T \delta(t)\left(\int_0^{y(t)} P_y(z)dz - \sum_{i=1}^{N} C_i(y_i, e_i, t) - D(e, S, t) \right) dt - \delta(T)F(S(T))$$

$$s.t.: \qquad \dot{S}(t) = e(t) - \gamma S(t) \tag{5}$$

$$y_i(t) \geq 0, \quad e_i(t) \geq 0$$

Differentiating the first order necessary conditions to the problem above yields a differential statement of the optimal emissions control path [8]

$$\frac{\partial C_i^*}{\partial a_i} - \frac{1}{(\rho + \gamma)} \frac{d}{dt}\left(\frac{\partial C_i^*}{\partial a_i} \right) = \frac{\partial D^*}{\partial e_i} + \frac{1}{(\rho + \gamma)}\left(\frac{\partial D^*}{\partial S_i} - \frac{d}{dt}\left(\frac{\partial D^*}{\partial e_i} \right) \right) \tag{6}$$

This equation shows that marginal abatement costs minus the present value of changes in marginal abatement costs through time should be equal to marginal damages from emissions plus the present value of marginal damages from an increase in the stock of pollution. This result is an extension of the result in Falk and Mendelsohn (1993:78), to the case where damages may depend on both emissions flows and stocks and the terminal value of emissions is considered. The question we now want to address is how to achieve the nationally (or internationally) optimal emission and stock path using flow permits.

4.0 Permit Banking and Borrowing for Individual Firms or Nations

Shown below in (7) is the individual nation's (or firm's) problem of maximizing GDP (profits) subject to emission constraints. At every point in time each nation (firm) is allocated emissions flow permits $\bar{e}_i$. These permits may be banked or borrowed subject to the bank equation of motion. Nations (firms) may also purchase or sell permits for pollution at the price P_e. The bank balances must be nonnegative at the terminal time T.

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[8] The single asterisks indicates that all the variables are evaluated at their collectively optimal levels. Here we rewrite marginal abatement costs $-\partial C_i/\partial e_i$ as $\partial C_i/\partial a_i$ for readability, defining marginal abatement da_i as marginal emissions reduction ($-de_i$).

$$J_i^{**} = \underset{y_i, e_i, x_i}{Max} \int_0^T \delta \{ P y_i - C_i(y_i, e_i, t) - P_e x_i \} dt$$

$$\begin{aligned} s.t: \quad & \dot{S}_i = e_i - \gamma S_i \\ & \dot{B}_{e_i} = \bar{e}_i - e_i + x_i \\ & x_{\min_i} \leq x_i \leq x_{\max_i} \\ & y_i \geq 0, \quad e_i \geq 0, \quad B_{e_i}(T) \geq 0 \end{aligned} \tag{7}$$

Solving the first order necessary conditions and rearranging as above yields the following expression:[9]

$$\frac{\partial C_i^{**}}{\partial a_i} - \frac{1}{(\rho + \gamma)} \frac{d}{dt}\left(\frac{\partial C_i^{**}}{\partial a_i} \right) = P_e^{**} - \frac{1}{\rho + \gamma}\left(\frac{d}{dt} P_e^{**} \right). \tag{8}$$

It is optimal for the nation or firm to expand emissions until the current marginal abatement costs minus the present value of changes in marginal abatement costs are equal to the price of a flow pollution permit minus the present value of future changes in the price of flow pollution permits. Here the present value calculation is based on an infinitely lived annuity which declines at the decay rate γ and is discounted at rate ρ. This simply says that with unrestricted banking and borrowing individual agents will adjust their marginal abatement costs until they equal permit prices at every point in time:

$$\frac{\partial C_i^{**}}{\partial a_i} = P_e^{**} \tag{9}$$

Since there is trading in each period, all firms face the same permit prices. The permit prices are, however, not independent across time. When firms have non-bounded solutions, then the following market outcome for permit price paths can be derived from differentiating and manipulating the first order conditions.

$$\begin{aligned} \dot{P}_e^{**} &= \rho P_e^{**} \\ \frac{\dot{P}_e^{**}}{P_e^{**}} &\equiv \hat{P}_e^{**} = \rho \end{aligned} \tag{10}$$

We now see that when firms are allowed to freely borrow and bank flow permits through time, on a one-to-one basis (a unitary intertemporal exchange rate), market permit prices (and marginal control costs) will rise at the rate of discount. Given unrestricted and interest-free banking and borrowing, agents will arbitrage permits across time until the discounted permit prices are equalized.

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[9] The double asterisks indicates that all the variables are evaluated at the non-cooperatively optimal levels for individual nations or firms.

To insure that the behavior of each agent (firm or government) conforms to the national or international optimum, the marginal abatement decisions by each agent must be the same as those expressed by the nationally or internationally optimal decision rule shown in ?. Unfortunately, this effort will be frustrated by the market arbitrage outcome which requires permit prices to rise at the discount rate (10). Alternatively, if banking and borrowing are prohibited, then permit prices will fluctuate each period depending on each period's permit endowment and marginal abatement costs. These yearly permit price fluctuations will also not, unless by accident, yield the correct intertemporal path for emissions.

5.0 Optimal Intertemporal Permit Trading Rates for Greenhouse Gas Emissions

Consider now that rather than allowing permits to trade on a one-to-one basis through time, that some exchange rate is applied whereby permits do not have the same value when used or saved in different periods. Altering the exchange rate is equivalent to altering the rate of change in discounted permit prices for different time periods, and can, in principle, direct firms to borrow and bank at globally or nationally optimal rates. Of course the correct amount of permits must also be issued to get the *level* of permit prices correct. Since the number of permits allocated in each period is the result of negotiations and interpretations of the Kyoto accord, annual permit allocations are likely to diverge from the international optimum. By introducing a banking regime, and altering the trading ratio, the regulatory authority can help correct for non-optimal permit endowments in each period.

The simplest way to adjust intertemporal trading rates is to include "interest" on permit bank account balances. Since bank account balances can be positive (saving) or negative (borrowing), a positive interest rate would reward saving and discourage borrowing. It would also imply that one permit saved now could be exchanged for more than one permit later. It is simple to include the "interest" payment or charge in the emission flow permit bank account dynamic equations:

$$\dot{B}_{e_i} = \bar{e}_i - e_i + x_i + r_e B_{e_i} \tag{11}$$

This alteration of the bank account equation of motion leads to the same optimality conditions as the even-exchange trading and banking case except for those conditions related to the time-rate of change of the shadow price of bank accounts. This means that all the previous results apply except that the time path of market permit prices is altered. The new percentage rates of change (indicated by a "hat" (^) symbol) of the permit prices are now:

$$\frac{\dot{\lambda}_e}{\lambda_e} = \frac{\dot{P}_e}{P_e} \equiv \hat{P}_e = \rho - r_e \tag{12}$$

Here we see that for firms to have a non-bounded, internal solution, permit prices must grow at the rate of discount *less* the rate of interest charged or paid on borrowed or banked emissions. Thus, the effect of a positive interest rate is to offset the discount rate and reduce the growth rate of market permit prices. This means that present value marginal abatement costs will decline through time relative to the zero-interest case. The only way for this to happen, ceteris paribus, is for emissions to increase through time faster than they would have with one-to-one intertemporal permit trading. Thus, an effect of paying positive "interest" on bank holding is, as one would suspect, to encourage extra emission reductions early in the T period time horizon.

Social optimality can be achieved under this system if, at every point in time, private emissions, e_i^{**}, (or marginal abatement costs) are identically equal to the socially optimal emissions, e_i^{*}, (abatement costs) for every firm (nation). This is true when the left-hand-sides of (8) and (6) are equal. Accordingly, their right-hand-sides should be equal as well. Thus, optimal permit prices should equal optimal marginal damages and, by choosing the trading ratio correctly, the percentage change in permit prices should equal the percentage change in marginal damages.

The question we would like to answer is how should the interest rate on bank accounts be set? For the GHG case considered here, where there are no flow damages, the cooperative optimality condition is:

$$\frac{\partial C_i^*}{\partial a_i} - \frac{1}{(\rho+\gamma)}\frac{d}{dt}\left(\frac{\partial C_i}{\partial a_i^*}\right) = \frac{1}{(\rho+\gamma)}\frac{\partial D^*}{\partial S_i}. \tag{13}$$

This implies a control path solution of the form:

$$\begin{aligned}\frac{\partial C^*}{\partial a_i} &= \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^*(\tau)}{\partial S_i}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F(T)}{\partial S} \\ &\equiv f_S^*(t).\end{aligned} \tag{14}$$

In words, this says that at any time in the planning horizon, the collectively optimal emission level is chosen such that discounted marginal abatement costs for each firm equals the present discounted value of all future marginal stock damages over the planning horizon plus the present value of marginal terminal stock damages which occur beyond the regulatory time horizon. Note that the "discount" rate used is (ρ+γ), the financial discount rate plus the stock decay rate.

In the case of flow-only permits, the market trading outcome would yield private abatement to the extent that at every point in time each firm's marginal abatement costs equals the price of a flow permit: $-C_{e_i}^{**} = P_e^{**}$, see (8). This is the usual static result. The regulatory authority, therefore, can induce firms to control their emission in an optimal manner by choosing $C_{e_i}^{**} = C_{e_i}^{*} = P_e^{*}$, where P_e^{*} reflects the present value of future marginal stock damages:

$$P_e^* = \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D(\tau)}{\partial S_i}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F(T)}{\partial S_i} \tag{16}$$

Taking the time derivative, the optimal permit price path ought to be:

$$\dot{P}_e^*(t) = -\frac{\partial D(t)}{\partial S} + (\rho+\gamma)P_e(t). \tag{17}$$

The optimal growth rates for flow permit prices, therefore, depend on the discount rate, the stock decay rates, and the marginal stock damage at every point in time.

For a permit trading and banking system, permit prices are not set or administered directly, but rather are a market outcome in response to the total number of permits allocated and the established banking rules, particularly the banking interest rate. The regulatory authority, in seeking optimality, should set the banking interest rate to assure coincidence of the market outcome and collectively optimal permit price paths, assuming that the starting permit price, as determined by the integral over time of all permit allocations, is optimal. This means that

$$\rho - r_e^* = \rho + \gamma - \frac{1}{P_e^*}\frac{\partial D^*}{\partial S}. \qquad (18)$$

Substituting in for P^*_e from (16) the optimal intertemporal trading rate, r_e*, for flow permits used to control damages from stock pollutants such as CO_2 is given by:

$$r_e^* = \frac{\dfrac{\partial D^*}{\partial S}}{f_S^*} - \gamma \qquad (19)$$

$$\text{Where } f_S^*(t) \equiv \int_t^T e^{-(\rho+\gamma)(\tau-t)}\frac{\partial D^*(\tau)}{\partial S(\tau)}d\tau + e^{-(\rho+\gamma)(T-t)}\frac{\partial F^*(T)}{\partial S(T)}$$

Interestingly, we see that the optimal intertemporal trading rate equals the ratio of *current* marginal stock damages to the *discounted future value* of marginal stock damages less the decay rate of emissions in the atmosphere. Each of these factors varies with the level of stock emissions. These factors may also vary with technical advances in damage mitigation, change in population, and changes in ecosystem resiliency due to other stresses.[10]

Permit Banking Rates When Private and Social Discount Rates Differ

The preceding results have not distinguished between the discount rates that may be used by individual agents (be they firms or governments) and the collective international planner. For the purposes of planning a banking system, the key point is that it is the *private* discount rate which will determine the time path of permit prices (through private arbitrage in permit markets), while it is the *social* discount rate which should be used in determining optimal abatement costs and marginal damages. Suppose that the private discount rate, i, exceeds the social rate ρ. With unrestricted banking and borrowing, market permit prices will grow at the private discount rate minus the flow permit banking interest rate r_e:

[10] The social discount rate ρ appears to cancel out of the efficient banking design condition in (18), but still appears implicitly in the future damage term f_S^*. The sensitivity of the banking interest rate to the discount rate remains to be shown numerically.

$$\hat{P}_e^{**} = i - r_e \tag{20}$$

In this case, the optimal banking interest rate or intertemporal trading rate is given by the condition:

$$i - r_e^* = -\frac{\frac{\partial D^*}{\partial S}}{f_S^*} + (\rho + \gamma). \tag{21}$$

When the private and social (individual agent and collective group) discount rates diverge, the flow permit banking interest rate must be increased by their difference, i-ρ:

$$r_e^* = \frac{\frac{\partial D^*}{\partial S}}{f_S^*} - \gamma + (i - \rho) \tag{22}$$

This simple but powerful extension has important implications for public policy. We can, for example, estimate the optimal banking rate corresponding to the case where current marginal stock damages are essentially zero. This serves as a lower bound on the optimal banking rate, since even though current damages may be small, it is not so clear that they are trivially small compared to the net present value of future marginal stock damages (i.e., that $(\partial D^*/\partial S)/f_S^* \ll 1\%$).

6.0 Numerical Estimation of Flexible Greenhouse Gas Emission Banking Systems

Back-of-the-Envelope Numerical Estimates of the Banking Interest Rate

We begin with a rough back-of-the-envelope estimate of the lower bound for the banking interest rate. To estimate the banking rate r_e, we need estimates of decay rate γ and the public and private discount rates. The residence time of CO_2 in the atmosphere depends on the rates of various biological and geophysical sinks (Trenberth 1992:218), and is sometimes represented by a detailed model rather than a fixed rate (Houghton *et al*., 1996:121). As an approximation, we can turn to the figure used by Nordhaus (1994:192, 1996), a decay factor of 8.33%/decade, or 0.8% per year.[11] For an exposition of the difficulties in establishing discount rates we can turn to the work of the Intergovernmental Panel on Climate Change (Arrow et al. 1996:131-133). Their balanced review of the literature presents rates for high-income industrial countries and also for developing countries. They find that equities have yielded a real rate of return of 5%, after accounting for taxation, or 7% pre-tax for many decades. The private (producer) discount rate would be

[11]This corresponds to a lifetime of 120 years. For greenhouse gas analysis, the lifetime is defined as the period over which the gas concentration falls to 1/e of its initial level. That is the "e-folding time," see Nordhaus, p. 26.

expected to be at this pretax level, or possibly much higher, for some projects.[12] Commonly used estimates of the social discount rate (social rate of time preference) reported in Arrow et al. range between 1% and 3%. This implies, therefore, that r_e^* could be in the range of 3%-5% per year, even when current marginal stock damages are essentially zero. This "lower bound" banking rate only accounts for the possible differences between public and private discount rates, and the decay rate of GHG stocks.

$$\begin{aligned} r_e^* &\geq (i - \rho) - \gamma \\ &\approx 4\% - 7\% \end{aligned} \tag{23}$$

Note that if the private/individual agent discount rate i equals the social/collective discount rate ρ, then all we can be assured of is that the optimal interest rate is no more negative than the stock decay rate, i.e, $r_e^* > -0.8\%$.

As another simplifying approximation, suppose now that the optimal marginal stock damages follows a smooth (exponential) growth path, i.e.

$$\frac{\partial D^*(\tau)}{\partial S(\tau)} = e^{g_D^*(\tau - t)} \frac{\partial D^*(t)}{\partial S(t)} \tag{24}$$

Here g_D^* is the growth rate of marginal damages along the optimal path, that is, in our earlier notation for the logarithmic time derivative, and using D_S to denote the derivative of damaged D with respect to stock S:

$$\frac{\dot{D}_S(t)}{D_S(t)} \equiv \hat{D}_S(t) \equiv g_D^* \quad \forall t \tag{25}$$

We maintain an asterisk on the optimal growth rate g_D^* to remind us that this rate is an optimizing outcome, *not* a constant of the system. Thus many factors, including the social discount rate, rates of technological change, marginal control costs, etcetera, are embedded in g_D^*. So far, we make no assumption about whether the growth rate g_D^* of marginal damages is positive or negative. However, *if* some approximately smooth growth rate applies, we can simplify the expression for the efficient banking interest rate:

[12] For GHG banking, a key issue would be which entities are allowed to make the permit borrowing/banking decisions (nation states, emitting firms, or speculators and traders), since that will have some bearing on the appropriate "private" discount rate for the analysis.

$$
\begin{aligned}
f_S^*(t) &= \int_t^T e^{-(\rho+\gamma)(\tau-t)} e^{g_D^*(\tau-t)} \frac{\partial D^*(t)}{\partial S(t)} d\tau + e^{-(\rho+\gamma)(T-t)} \int_T^\infty e^{-(\rho+\gamma)(\tau-T)} e^{g_D^*(\tau-T)} \frac{\partial D^*(T)}{\partial S(T)} d\tau \\
&= \frac{\partial D^*(t)}{\partial S(t)} \int_t^\infty e^{(g_D^*-(\rho+\gamma))(\tau-t)} d\tau \\
&= \frac{\partial D^*(t)}{\partial S(t)} \left. \frac{e^{(g_D^*-(\rho+\gamma))(\tau-t)}}{g_D^*-(\rho+\gamma)} \right|_t^\infty \\
&= \frac{\partial D^*(t)}{\partial S(t)} \frac{1}{(\rho+\gamma)-g_D^*} \quad \text{if } g_D^*<(\rho+\gamma)
\end{aligned}
\tag{26}
$$

This is the net present value of a stream of marginal stock damages for which damages per unit stock grow at rate g_D^* but are discounted at rate ρ and the stock decays at rate γ.[13] Substituting this expression for the future stock damages into the expression for the optimal banking interest rate (22), we conclude

$$
\begin{aligned}
r_e^* &= \frac{\frac{\partial D^*}{\partial S}}{\left(\frac{\frac{\partial D^*}{\partial S}}{(\rho+\gamma)-g_D^*} \right)} - \gamma + (i-\rho) \\
&= (\rho+\gamma)-g_D^* - \gamma + (i-\rho) \\
&= i - g_D^*
\end{aligned}
\tag{27}
$$

The optimal interest rate, from the social perspective, is simply the private (individual agent) discount rate minus the growth rate of marginal stock damages along the optimal path. We would expect the optimal growth rate of marginal stock damages to be small, since the stock changes slowly. Hence we expect the optimal banking interest rate to be near the discount rate, specifically the "private" or individual-agent discount rate, if that differs from the collective or social discount rate.

Note that this estimate (27), while appearing to depend on very different terms, is entirely consistent with the lower bound estimate of (23). As mentioned above, for the solution to be well defined (bounded damages), it must be true that the optimal growth rate of marginal damages be less than the social discount rate plus the stock decay rate ($g_D^*<\rho+\gamma$). So we see that the approximation in (27) is above the lower bound:

$$
i - g_D^* \geq (i-\rho) - \gamma \tag{28}
$$

[13]It is perfectly reasonable to presume that the optimal growth rate of marginal stock damages is less than the discount rate plus the stock decay rate, because otherwise future marginal damages of a unit of stock now would be unbounded, hardly an optimal planning outcome!

Refined Numerical Estimates Using Nordhaus/Falk and Mendelsohn Based Model

Falk and Mendelsohn (1993) develop a continuous-time optimal control model which minimizes the present value of pollution damages and abatement costs over a T-period time horizon to derive optimal abatement paths through time. Their parameters for control costs, damage functions, and other climatological relationships draw heavily on Nordhaus (1991). Since they do not use final value function, and assume a free terminal value, they obtain the following transversality condition, showing that the shadow cost of terminal stock is zero, hence the terminal period marginal abatement cost should be zero: $\Lambda(T) = 0 = -rT\frac{\partial C(T)}{\partial a}$. In contrast, Manne (1986) in his GAMS implementation of Ramsey's (1928) model of savings, shows a way to get around the terminal value problem by weighting the final period benefits more heavily, corresponding to the presumption that those benefits will be maintained as a steady state outcome.[14]

In Appendix 1 we display some computer code using the programming language GAMS (General Algebraic Modeling System) to estimate a banking and borrowing permit system based on the work of Falk and Mendelsohn (1993).[15] We used this code to validate our analytic results

---

[14] The multiplicative factor 1/(1-δ) applied to the terminal period discount factor corresponds to the net present value of an perpetuity discounted with the discount factor δ. The combined discount factor $\delta^T/(1-\delta)$ gives the net present value of a perpetuity beginning at time *T*:

$$\delta = \frac{1}{1+r}$$

$$\sum_{t=T}^{\infty} \frac{1}{(1+r)^t} = \sum_{t=T}^{\infty} \delta^t = \frac{\delta^T}{1-\delta}$$

$$\text{and, } \frac{\delta^T}{(1-\delta)} = \frac{1}{(1+r)^T}\cdot\frac{(1+r)}{r}.$$

Note that this approach weights the terminal period "intermediate value" net benefits function rather than the capital terminal stock.

$$J \equiv \sum_{t=0}^{T} \delta(t)\Big(D_1(t)S(t) + D_2S(t) + C\cdot A(T)^2\Big) + F(S(T),E(T))$$

$$\begin{aligned} E(t) &= E_U(t) - A(t), & \forall\ t \\ S(t+1) &= S(t)(1-g) + E(t), & \forall\ t \\ E(t) &\geq 0, & \forall\ t \end{aligned}$$

where

$$\delta(t) \equiv \begin{cases} \delta^t & t=0..T-1 \\ \dfrac{\delta^T}{1-\delta} & t=T \end{cases}$$

[15] Other models also conveniently available for further numerical experiments are those by Manne and Richels (1993), Nordhaus (1994), and Nordhaus and Yang (1996).

presented above, and to explore optimal banking regimes. The model was run to generate a collectively optimal emission abatement path, and that path was use to set a global emission permit allocation in each time period. We then modified the model to examine the privately optimal (non-cooperative) outcomes, under three conditions:

1. Unlimited permits (no restrictions);
2. Unrestricted banking/borrowing, with zero interest;
3. Banking and borrowing with a non-zero interest rate.

The two banking and borrowing cases were subject to the constraint that by the terminal time period all bank account balances must be zero. In the first case the private agents engaged in no emissions reduction. In the second case they borrowed heavily in the early periods (emitted more than the number of permits issued for the first 100 years), and paid back their accounts in the last 50 years. This indicates that a zero interest rate is certainly below optimal.

The third case is our general banking regime with a time-varying banking interest rate. To determine the interest rate for case 3, we observed the time rate of change of optimal marginal abatement cost, thereby determining the socially efficient time path for permit prices $\hat{P}_e^*$. For unrestricted permit banking with flow permit interest rate r_e, permits will be arbitraged over time until the growth rate of market permit P_e^{**} prices satisfies:

$$\hat{P}_e^{**} \equiv \frac{dlnP_e^{**}}{dt} = i - r_e \tag{29}$$

where i is the individual/private discount rate. If we know the desired time path of permit prices $\hat{P}_e^*$, then to achieve it, regulator should set r_e^* so that:

$$r_e^* = i - \hat{P}_e^* \tag{30}$$

Here we recognize that the private discount rate i may differ from the social rate ρ. If we know the time-rate-of-change of optimal social marginal abatement costs, $\hat{C}_a^*$, then we can determine the socially optimal interest rate on flow permit bank accounts, by setting the time-rate-of-change of flow permit prices, P_e, to match the time-rate-of-change of optimal social marginal abatement costs. That is, private agents will adjust their marginal control costs $\hat{C}_a^{e**}$, to track flow permit prices:

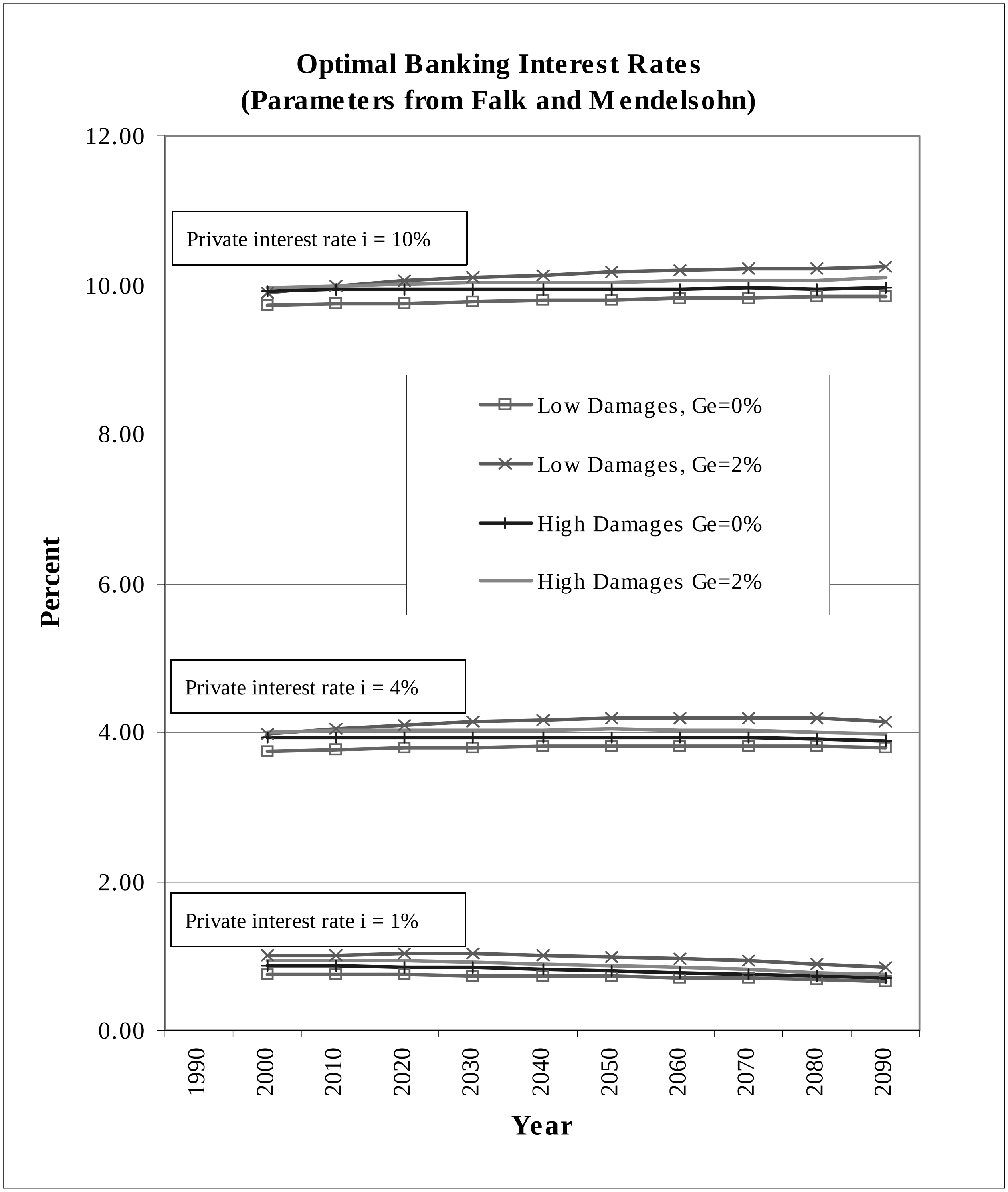
$$C_a^{**} = P_e, \quad \hat{C}_a^{**} = \hat{P}_e \tag{31}$$

We want, for social optimality, private abatement efforts to match the socially optimal level, e.g., $\hat{C}_a^{**} = \hat{C}_a^*$. To engineer the time path of market permit prices so that $\hat{P}_e^{**} = \hat{C}_a^*$, regulators should set

$$r_e^* = i - \hat{C}_a^* \,. \tag{32}$$

We confirmed that this banking interest rate does indeed cause private banking behavior to track the socially optimal path.

We then exercised the model over a range of assumptions regarding discount rates, GHG stock damages, and growth rates in GHG emissions. For the range of sensitivity cases considered, we found that optimal interest rates stay very close to the private discount rate, but are relatively stable with respect to changes the magnitude of damages or an autonomous increase in the emissions growth rate. (See the table below)


Optimal Banking Interest Rates
(Parameters from Falk and Mendelsohn)
Percent
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Private interest rate i = 10%
Private interest rate i = 4%
Private interest rate i = 1%
Low Damages, Ge=0%
Low Damages, Ge=2%
High Damages Ge=0%
High Damages Ge=2%
1990
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
Year

Numerical Results: Socially Optimal Banking Interest Rates

Lower Damages,
Autonomous Emissions Growth Rate = 0%

	i=1%	*i*=4%	*i*=10%
1990	0.010	0.040	0.100
1991	0.007	0.037	0.097
1992	0.007	0.037	0.097
1993	0.007	0.037	0.097
1994	0.007	0.037	0.097
1995	0.007	0.037	0.097
1996	0.007	0.037	0.097
1997	0.007	0.037	0.097
1998	0.007	0.037	0.097
1999	0.007	0.037	0.097
2000	0.007	0.037	0.097
2001	0.007	0.037	0.097
2002	0.007	0.038	0.097
2003	0.007	0.038	0.097
2004	0.007	0.038	0.097
2005	0.007	0.038	0.097
2006	0.007	0.038	0.097
2007	0.007	0.038	0.097
2008	0.007	0.038	0.097
2009	0.007	0.038	0.097
2010	0.007	0.038	0.097
2011	0.007	0.038	0.097
2012	0.007	0.038	0.097
2013	0.007	0.038	0.097
2014	0.007	0.038	0.098
2015	0.007	0.038	0.098
2016	0.007	0.038	0.098
2017	0.007	0.038	0.098
2018	0.007	0.038	0.098
2019	0.007	0.038	0.098
2020	0.007	0.038	0.098
2021	0.007	0.038	0.098
2022	0.007	0.038	0.098
2023	0.007	0.038	0.098
2024	0.007	0.038	0.098
2025	0.007	0.038	0.098
2026	0.007	0.038	0.098
2027	0.007	0.038	0.098
2028	0.007	0.038	0.098
2029	0.007	0.038	0.098
2030	0.007	0.038	0.098
2031	0.007	0.038	0.098
2032	0.007	0.038	0.098
2033	0.007	0.038	0.098
2034	0.007	0.038	0.098
2035	0.007	0.038	0.098
2036	0.007	0.038	0.098
2037	0.007	0.038	0.098
2038	0.007	0.038	0.098
2039	0.007	0.038	0.098
2040	0.007	0.038	0.098
2041	0.007	0.038	0.098
2042	0.007	0.038	0.098
2043	0.007	0.038	0.098
2044	0.007	0.038	0.098
2045	0.007	0.038	0.098
2046	0.007	0.038	0.098
2047	0.007	0.038	0.098
2048	0.007	0.038	0.098
2049	0.007	0.038	0.098
2050	0.007	0.038	0.098
2051	0.007	0.038	0.098
2052	0.007	0.038	0.098
2053	0.007	0.038	0.098
2054	0.007	0.038	0.098
2055	0.007	0.038	0.098
2056	0.007	0.038	0.098
2057	0.007	0.038	0.098
2058	0.007	0.038	0.098
2059	0.007	0.038	0.098
2060	0.007	0.038	0.098
2061	0.007	0.038	0.098
2062	0.007	0.038	0.098
2063	0.007	0.038	0.098
2064	0.007	0.038	0.098
2065	0.007	0.038	0.098
2066	0.007	0.038	0.098
2067	0.007	0.038	0.098
2068	0.007	0.038	0.098
2069	0.007	0.038	0.098
2070	0.007	0.038	0.098
2071	0.007	0.038	0.098
2072	0.007	0.038	0.098
2073	0.007	0.038	0.098
2074	0.007	0.038	0.098
2075	0.007	0.038	0.098
2076	0.007	0.038	0.098
2077	0.007	0.038	0.098
2078	0.007	0.038	0.098
2079	0.007	0.038	0.098
2080	0.007	0.038	0.098
2081	0.007	0.038	0.098
2082	0.007	0.038	0.098
2083	0.007	0.038	0.098
2084	0.007	0.038	0.098
2085	0.007	0.038	0.098
2086	0.007	0.038	0.098
2087	0.007	0.038	0.098
2088	0.007	0.038	0.098
2089	0.007	0.038	0.098
2090	0.007	0.038	0.098

Lower Damages,
Autonomous Emissions Growth Rate = 2%

	i=1%	*i*=4%	*i*=10%
1990	0.010	0.040	0.100
1991	0.010	0.039	0.098
1992	0.010	0.039	0.098
1993	0.010	0.039	0.098
1994	0.010	0.039	0.098
1995	0.010	0.039	0.099
1996	0.010	0.039	0.099
1997	0.010	0.040	0.099
1998	0.010	0.040	0.099
1999	0.010	0.040	0.099
2000	0.010	0.040	0.099
2001	0.010	0.040	0.099
2002	0.010	0.040	0.099
2003	0.010	0.040	0.099
2004	0.010	0.040	0.099
2005	0.010	0.040	0.099
2006	0.010	0.040	0.100
2007	0.010	0.040	0.100
2008	0.010	0.040	0.100
2009	0.010	0.040	0.100
2010	0.010	0.040	0.100
2011	0.010	0.040	0.100
2012	0.010	0.041	0.100
2013	0.010	0.041	0.100
2014	0.010	0.041	0.100
2015	0.010	0.041	0.100
2016	0.010	0.041	0.100
2017	0.010	0.041	0.100
2018	0.010	0.041	0.100
2019	0.010	0.041	0.100
2020	0.010	0.041	0.100
2021	0.010	0.041	0.101
2022	0.010	0.041	0.101
2023	0.010	0.041	0.101
2024	0.010	0.041	0.101
2025	0.010	0.041	0.101
2026	0.010	0.041	0.101
2027	0.010	0.041	0.101
2028	0.010	0.041	0.101
2029	0.010	0.041	0.101
2030	0.010	0.041	0.101
2031	0.010	0.041	0.101
2032	0.010	0.041	0.101
2033	0.010	0.041	0.101
2034	0.010	0.041	0.101
2035	0.010	0.041	0.101
2036	0.010	0.041	0.101
2037	0.010	0.041	0.101
2038	0.010	0.042	0.101
2039	0.010	0.042	0.101
2040	0.010	0.042	0.101
2041	0.010	0.042	0.101
2042	0.010	0.042	0.101
2043	0.010	0.042	0.101
2044	0.010	0.042	0.102
2045	0.010	0.042	0.102
2046	0.010	0.042	0.102
2047	0.010	0.042	0.102
2048	0.010	0.042	0.102
2049	0.010	0.042	0.102
2050	0.010	0.042	0.102
2051	0.010	0.042	0.102
2052	0.010	0.042	0.102
2053	0.010	0.042	0.102
2054	0.010	0.042	0.102
2055	0.010	0.042	0.102
2056	0.010	0.042	0.102
2057	0.010	0.042	0.102
2058	0.010	0.042	0.102
2059	0.010	0.042	0.102
2060	0.010	0.042	0.102
2061	0.010	0.042	0.102
2062	0.010	0.042	0.102
2063	0.010	0.042	0.102
2064	0.010	0.042	0.102
2065	0.009	0.042	0.102
2066	0.009	0.042	0.102
2067	0.009	0.042	0.102
2068	0.009	0.042	0.102
2069	0.009	0.042	0.102
2070	0.009	0.042	0.102
2071	0.009	0.042	0.102
2072	0.009	0.042	0.102
2073	0.009	0.042	0.102
2074	0.009	0.042	0.102
2075	0.009	0.042	0.102
2076	0.009	0.042	0.102
2077	0.009	0.042	0.102
2078	0.009	0.042	0.102
2079	0.009	0.042	0.102
2080	0.009	0.042	0.102
2081	0.009	0.042	0.102
2082	0.009	0.042	0.102
2083	0.009	0.042	0.102
2084	0.009	0.042	0.102
2085	0.009	0.042	0.102
2086	0.009	0.042	0.102
2087	0.009	0.042	0.102
2088	0.008	0.042	0.102
2089	0.008	0.042	0.102
2090	0.008	0.041	0.102

Higher Damages,
Autonomous Emissions Growth Rate = 0%

	i=1%	*i*=4%	*i*=10%
1990	0.010	0.040	0.100
1991	0.009	0.039	0.099
1992	0.009	0.039	0.099
1993	0.009	0.039	0.099
1994	0.009	0.039	0.099
1995	0.009	0.039	0.099
1996	0.009	0.039	0.099
1997	0.009	0.039	0.099
1998	0.009	0.039	0.099
1999	0.009	0.039	0.099
2000	0.009	0.039	0.099
2001	0.009	0.039	0.099
2002	0.009	0.039	0.099
2003	0.009	0.039	0.099
2004	0.009	0.039	0.099
2005	0.009	0.039	0.099
2006	0.009	0.039	0.099
2007	0.009	0.039	0.099
2008	0.009	0.039	0.099
2009	0.009	0.039	0.099
2010	0.009	0.039	0.099
2011	0.009	0.039	0.099
2012	0.009	0.039	0.099
2013	0.009	0.039	0.099
2014	0.009	0.039	0.099
2015	0.009	0.039	0.099
2016	0.009	0.039	0.099
2017	0.009	0.039	0.099
2018	0.009	0.039	0.099
2019	0.008	0.039	0.099
2020	0.008	0.039	0.099
2021	0.008	0.039	0.099
2022	0.008	0.039	0.099
2023	0.008	0.039	0.099
2024	0.008	0.039	0.099
2025	0.008	0.039	0.099
2026	0.008	0.039	0.099
2027	0.008	0.039	0.099
2028	0.008	0.039	0.099
2029	0.008	0.039	0.099
2030	0.008	0.039	0.099
2031	0.008	0.039	0.099
2032	0.008	0.039	0.099
2033	0.008	0.039	0.099
2034	0.008	0.039	0.099
2035	0.008	0.039	0.099
2036	0.008	0.039	0.099
2037	0.008	0.039	0.099
2038	0.008	0.039	0.099
2039	0.008	0.039	0.099
2040	0.008	0.039	0.099
2041	0.008	0.039	0.099
2042	0.008	0.039	0.099
2043	0.008	0.039	0.099
2044	0.008	0.039	0.099
2045	0.008	0.039	0.099
2046	0.008	0.039	0.099
2047	0.008	0.039	0.099
2048	0.008	0.039	0.099
2049	0.008	0.039	0.099
2050	0.008	0.039	0.099
2051	0.008	0.039	0.099
2052	0.008	0.039	0.099
2053	0.008	0.039	0.099
2054	0.008	0.039	0.099
2055	0.008	0.039	0.099
2056	0.008	0.039	0.099
2057	0.008	0.039	0.099
2058	0.008	0.039	0.099
2059	0.008	0.039	0.099
2060	0.008	0.039	0.099
2061	0.008	0.039	0.099
2062	0.008	0.039	0.099
2063	0.008	0.039	0.099
2064	0.008	0.039	0.099
2065	0.008	0.039	0.099
2066	0.008	0.039	0.100
2067	0.008	0.039	0.100
2068	0.008	0.039	0.100
2069	0.008	0.039	0.099
2070	0.008	0.039	0.100
2071	0.008	0.039	0.100
2072	0.007	0.039	0.100
2073	0.007	0.039	0.100
2074	0.007	0.039	0.100
2075	0.007	0.039	0.100
2076	0.007	0.039	0.100
2077	0.007	0.039	0.100
2078	0.007	0.039	0.100
2079	0.007	0.039	0.100
2080	0.007	0.039	0.100
2081	0.007	0.039	0.100
2082	0.007	0.039	0.100
2083	0.007	0.039	0.099
2084	0.007	0.039	0.100
2085	0.007	0.039	0.099
2086	0.007	0.039	0.099
2087	0.007	0.039	0.099
2088	0.007	0.039	0.100
2089	0.007	0.039	0.099
2090	0.007	0.039	0.100

Higher Damages,
Autonomous Emissions Growth Rate = 2%

	i=1%	*i*=4%	*i*=10%
1990	0.010	0.040	0.100
1991	0.009	0.040	0.100
1992	0.009	0.040	0.100
1993	0.009	0.040	0.100
1994	0.009	0.040	0.100
1995	0.009	0.040	0.100
1996	0.009	0.040	0.100
1997	0.009	0.040	0.100
1998	0.009	0.040	0.100
1999	0.009	0.040	0.100
2000	0.009	0.040	0.100
2001	0.009	0.040	0.100
2002	0.009	0.040	0.100
2003	0.009	0.040	0.100
2004	0.009	0.040	0.100
2005	0.009	0.040	0.100
2006	0.009	0.040	0.100
2007	0.009	0.040	0.100
2008	0.009	0.040	0.100
2009	0.009	0.040	0.100
2010	0.009	0.040	0.100
2011	0.009	0.040	0.100
2012	0.009	0.040	0.100
2013	0.009	0.040	0.100
2014	0.009	0.040	0.100
2015	0.009	0.040	0.100
2016	0.009	0.040	0.100
2017	0.009	0.040	0.100
2018	0.009	0.040	0.100
2019	0.009	0.040	0.100
2020	0.009	0.040	0.100
2021	0.009	0.040	0.100
2022	0.009	0.040	0.100
2023	0.009	0.040	0.100
2024	0.009	0.040	0.100
2025	0.009	0.040	0.100
2026	0.009	0.040	0.100
2027	0.009	0.040	0.100
2028	0.009	0.040	0.100
2029	0.009	0.040	0.100
2030	0.009	0.040	0.100
2031	0.009	0.040	0.100
2032	0.009	0.040	0.100
2033	0.009	0.040	0.100
2034	0.009	0.040	0.100
2035	0.009	0.040	0.100
2036	0.009	0.040	0.100
2037	0.009	0.040	0.100
2038	0.009	0.040	0.100
2039	0.009	0.040	0.100
2040	0.009	0.040	0.100
2041	0.009	0.040	0.100
2042	0.009	0.040	0.100
2043	0.009	0.040	0.100
2044	0.009	0.040	0.100
2045	0.009	0.040	0.100
2046	0.009	0.040	0.100
2047	0.009	0.040	0.100
2048	0.009	0.040	0.100
2049	0.009	0.040	0.100
2050	0.009	0.040	0.100
2051	0.009	0.040	0.100
2052	0.009	0.040	0.100
2053	0.009	0.040	0.100
2054	0.009	0.040	0.100
2055	0.009	0.040	0.100
2056	0.009	0.040	0.100
2057	0.009	0.040	0.100
2058	0.008	0.040	0.100
2059	0.008	0.040	0.100
2060	0.008	0.040	0.100
2061	0.008	0.040	0.100
2062	0.008	0.040	0.100
2063	0.008	0.040	0.100
2064	0.008	0.040	0.101
2065	0.008	0.040	0.101
2066	0.008	0.040	0.101
2067	0.008	0.040	0.101
2068	0.008	0.040	0.101
2069	0.008	0.040	0.101
2070	0.008	0.040	0.101
2071	0.008	0.040	0.101
2072	0.008	0.040	0.101
2073	0.008	0.040	0.101
2074	0.008	0.040	0.101
2075	0.008	0.040	0.101
2076	0.008	0.040	0.101
2077	0.008	0.040	0.101
2078	0.008	0.040	0.101
2079	0.008	0.040	0.101
2080	0.008	0.040	0.101
2081	0.008	0.040	0.101
2082	0.008	0.040	0.101
2083	0.008	0.040	0.101
2084	0.008	0.040	0.101
2085	0.008	0.040	0.100
2086	0.008	0.040	0.101
2087	0.008	0.040	0.100
2088	0.007	0.040	0.101
2089	0.007	0.040	0.100
2090	0.007	0.040	0.101

7.0 Conclusions and Future Research

We have developed the theoretical basis for establishing an intertemporal exchange rate, or "banking interest rate," for banking and borrowing permits to emit a stock pollutant. With some simplifications, the expression for the banking interest rate that promotes collectively efficient banking behavior by individual agents can be reduced to a very few simple terms. Key determinants are the private and social discount rates, the stock decay rate, and the growth rate of marginal stock damages along the socially optimal path. Furthermore, we can establish a lower bound on the banking interest rate based on exogenous parameters: the difference between the private ane public discount rate minus the stock decay rate. The efficient banking interest rate *could* be negative, but no more negative in magnitude than the stock decay rate. For GHGs, this is a small number (~ -1%).

Our intent, is to apply the theoretical constructs developed here to existing numerical models. While we have worked out the basic theoretic aspects of stock-pollutant banking analytically, further numerical experiments will allow us to validate our analytical insights and explore the magnitude of permit banking consequences, optimal design and climate-economic benefits. Specifically, the policy issues to address numerically are:

- How to use intertemporal exchange rates to implement permit banking in the context of second-best (negotiated) emission reduction agreements;
- How changes in permit allocations and emission targets affect the economic and climatological merits of GHG permit banking;
- How sensitive efficient banking design is to key parameters such as growth rates in emissions per unit GDP, abatement costs, and damages.
- How differences in discount rates among national governments and private agents affect the performance and design of emission banking system.

We fully recognize the limits imposed by the simplifications necessary for a compact analytical expression of global permit trading. Numerical models will require fewer simplifications, but will still only be broad approximations. Nonetheless, we feel that intertemporal permit trading is one of the most powerful tools available to help control GHGs, and worthy of further investigation.

Appendix 1

Sample Computer Code Adapting the Falk and Mendelsohn (1993) Model Using the GAMS (General Algebraic Modeling System) Programming Language.

SUBSCRIPTS

t,τ	index over time periods
T	Final time period

SCALARS

E_0	Initial Gross Emissions	/7.92 /
S_0	Initial Pollutant Stock	/800 /
D_1	Damage Fn Linear Coeff	/-0.0325 /
D_2	Damage Fn Quadratic Coeff	/4.06E-5 /
C_2	Abatement Cost Fn Quadratic Coeff	/18 /
g	Stock Decay Rate	/0.005 /
I	Discount Rate	/0.01 /
α	Damage Nonautonomous Growth Rate	/0 /
β	Gross Emissions Nonautonomous Growth Rate	/0 /
θ	Abatement Cost Nonautonomous Growth Rate	/0 /
$\underline{D}_F$	Future Damages per Unit Pollutant Stock	/0.1/
Y	Time Step (Years per Discrete Time Step/Period)	/1/

VARIABLES {unrestricted in sign}

$B_E(t)$	"Emission Flow Permit Bank Account"
$B_S(t)$	"Emission Stock Permit Bank Account"
D_F	"Final (Terminal) Valuation/Damages of Durable Stock"
J	"Objective Function Value, NPV Costs ($ Bill)";

POSITIVE VARIABLES

A(t)	"Emission Abatement"
E(t)	"Emissions After Abatement"
S(t)	"Pollutant Stock"
D(t)	"Damage/losses Due to Pollutant Stock"
C(t)	"Abatement or Control Costs"

PARAMETERS

$E_U(t)$	"Gross Unabated Emissions (Bill Tons)"
$\delta(t)$	"Discount Factor"
$\underline{\delta}(t)$	"Discount Factor with term for infinite stream in terminal period"
$\underline{\delta}(T)$	"Discount Factor for terminal period (T+1?) infinite stream"
MC(t)	"Marginal Abatement Cost ($/ton)"

MD(t) "Marginal Damages ($/ton)"
MMC(t) "Second Derivative of Abatement Cost fn"
APCT(t) "Abatement as Percent of Emissions"
$P_E(t)$ "Socially Opt Emission Permit Price"
$P_S(t)$ "Socially Opt Stock Permit Price"
$r_e(t)$ "Banking Interest Rate for Emission Flow Permits"
$r_S(t)$ "Banking Interest Rate for Emission Stock Permits"
$\underline{E}(t)$ "Emission Permit Levels"
$\underline{S}(t)$ "Stock Permit Levels"
$\Delta\underline{S}(t)$ "Change in Stock Permit Levels"

PARAMETERS { for reporting across cases }
ROBJ(*) "Objective Value"
RDISCFACTT(T,*) "Discount Factor with Infintite Stream Terminal Period"
RDISCLAST(*) "Discount Factor for T+1 Period Damages Taken Account of at Time t"
RA(t,*) "Emission Abatement"
RE(t,*) "Emissions After Abatement"
RS(t,*) "Pollutant Stock"
RMD(t,*) "Marginal Damage Due to Pollutant Stock"
RMC(t,*) "Marginal Abatement or Control Costs"
RBE(t,*) "Bank Account For Emissions Flow Permits"
RBS(t,*) "Bank Account For Emissions Stock Permits"
RMDMC(T,*) "Difference Between MD and MC, MD-MC"
RSLAST(*) "Stock in T+1"

Parameter Initializations
$E_U(t) = E_0\, e^{\beta tc}$ {unabated emissions grow at rate BETA}
$*\delta(t) = e^{-it}$ {contin-time disc factor for each period}
$\delta(t) = (1+i)^{-t}$ {discrete-time disc factor for each period}
$P_E(t) = 0$ { initial emission permit price zero }
$P_S(t) = 0$ { initial stock permit price zero }
$r_e(t) = 0$ { initial interest rate on flow permit bank balance is zero }

EQUATIONS

EEQ(t) "EMISSIONS AFTER ABATEMENT EQUATION (Bill. Tons)"
Emissions E are unabated emissions EU less abatement A.

$$E(t) = E_U(t) - A(t)$$

DEQ(t) "DAMAGE DUE TO POLLUTANT STOCK ($ Bill)"
Quadratic stock pollutant damages.

$$D(t) = D_1 S(t) + D_2 S(t)^2$$

CEQ(t) "ABATEMENT OR CONTROL COSTS ($ Bill)"
Purely quadratic abatement costs.

$$C(t) = C_2 A(t)^2$$

FEQ(t) "RECURSIVE FINAL PERIOD STOCK VALUATION DEF"
Salvage value for accumulated capital.

$$FINALVAL(T) = \delta(T) S(T) F_V$$

{ Stock Evolution }
SEQ(t) "POLLUTANT STOCK EVOLUTION (Bill. Tons)"
Stock evolution with decay, new emissions after abatement.

$$S(t+1) = S(t)(1-g) + E(t)$$

EBEQ(t) "EMISSIONS FLOW BOUNDS CONSTRAINT EQUATION"
Emissions flow limit equation.

$$E(t) \leq \underline{E}(t)$$

SBEQ(t) "EMISSIONS STOCK BOUNDS CONSTRAINT EQUATION"
Emissions stock limit equation.

$$S(t) \leq \underline{S}(t)$$

{ Emission Flow Bank Account Evolution }
BEEQ(t) "BANK OF FLOW PERMITS EVOLUTION (Bill. Tons)"
Stock evolution with new permits less new emissions.

$$B_E(t+1) = B_E(t)(1+r_e(t)) + \underline{E}(t) - E(t)$$

{ Emission Stock Bank Account Evolution - Permanent Stock Permits }
BSPERMEQ(t) "BANK OF PERMANENT STOCK PERMITS EVOLUTION (Bill. Tons)"
Bank evolution with new permits, less new emissions, plus decay

$$B_S(t+1) = B_S(t) + \Delta\underline{S}(t) - (E(t) - S(t)g)$$

{ Emission Stock Bank Account Evolution - Temp Stock Permits }
BSTEMPEQ(t) "BANK OF TEMPORARY STOCK PERMITS EVOLUTION (Bill. Tons)"

Bank evolution with new permits, less new emissions, plus decay.

$$B_S(t+1) \;=\; B_S(t)(1+r_S(t)) + \underline{S}(t) - S(t)$$

{ Emission Flow Bank Account Final Condition }

BEFINALEQ(T) "FINAL BOUND ON BANK ACCT OF EMISSION FLOW PERMITS"

$$B_E(T) \;\geq\; 0$$

{ Emission Stock Bank Account Final Condition }

BSFINALEQ(T) "FINAL BOUND ON BANK ACCT OF EMISSION STOCK PERMITS"

$$B_S(T) \;\geq\; 0$$

SOBJEQ "SOCIAL OBJECTIVE FUNCTION DEFINITION"

SOBJEQ.. {NPV of Social damage and control costs}

This includes the sum over time of discounted quadratic stock pollutant damages and purely quadratic abatement costs, plus the Salvage value for accumulated terminal stock>

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t)\Big(D_1 S(t) \;+\; D_2 S(t)^2 \;+\; C_2 A(T)^2\Big) \;+\; \delta(T)S(T)F_V$$

POBJEQ "PRIVATE OBJECTIVE FUNCTION DEFINITION"

NPV of control costs and permit costs.

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t)C_2 A(t)^2$$

Sum over time and region of purely quadratic abatement costs.

PTAXOBJEQ "PRIVATE OBJECTIVE FUNCTION DEFINITION WITH TAXES"

NPV of control costs and permit costs.

$$J \;\equiv\; \sum_{t=0}^{T} \delta(t)\Big(P_S(t)S(t) \;+\; P_E(t)E(t) \;+\; C_2 A(T)^2\Big) \;+\; \delta(T)S(T)F_V$$

This includes, for each period, payments for stock permits, payments for emission permits, and purely quadratic abatement costs. In the final year it also includes the Salvage value for accumulated capital.

Model Declarations

Model names are declared and then assigned equations by including equation names.

The social optimum model minimizes social damages plus control costs, where control costs depend on abatement effort, and damages _could_ depend on both emission flows and stocks.

The constraints on cost minimization are:

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

```
MODEL SOCIALOPT   SOCIAL OPTIMUM STOCK POLLUTANT MODEL
     /
    EEQ, SEQ, SOBJEQ
     /;
```

The private optimum model minimizes private damages plus control costs, where control costs depend on abatement effort, and private damages are zero.

The constraints on cost minimization are:

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVOPT     PRIVATE DECISION UNCONSTRAINED
     /
    EEQ, SEQ, BEEQ, BSEQ, POBJEQ
     /;
```

The private permit model minimizes private damages plus control costs, given emission and stock permit levels and no banking/borrowing. Here control costs depend on abatement effort, and private damages are zero.

The constraints on cost minimization are

emission flows and stocks bounded by permits in each period (EBEQ,SBEQ),

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVPERM    PRIVATE DECISION GIVEN PERMITS AND NO BANKING
     /
    EBEQ, SBEQ,
    EEQ, SEQ, BEEQ, BSEQ, POBJEQ
     /;
```

The private banking model minimizes private damages plus control costs, given emission and stock permit levels and banking/borrowing. Here control costs depend on abatement effort, and private damages are zero.

The constraint on cost minimization are

emission flow and stock bank account must be positive in final period,

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

For comparison purposes, the bank accounts for emission flows and stocks are also tracked

(BEEQ, BSEQ).

```
MODEL PRIVBANK    PRIVATE DECISION GIVEN PERMITS AND BANKING
    /
   BEFINALEQ, BSFINALEQ, {only require final bank balances positive}
   EEQ, SEQ, BEEQ, BSEQ, POBJEQ
    /;
```

The private optimum model minimizes private damages plus control costs, plus the payments for emission or stock fees/taxes. Here control costs depend on abatement effort, and private damages are zero.

The constraint on cost minimization are

the relation between emissions level and abatement level (EEQ), and

the pollutant stock evolution equation (SEQ).

For comparison purposes, the bank accounts for emission flows and stocks are also tracked (BEEQ, BSEQ).

```
MODEL PRIVTAX     PRIVATE DECISION UNCONSTRAINED
    /
   EEQ, SEQ, BEEQ, BSEQ, PTAXOBJEQ
    /;
```

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