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TECHNICAL DOCUMENTATION OF THE TRANSITIONAL ALTERNATIVE FUELS AND VEHICLES (TAFV) MODEL[1]

Model Version 1.0

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LIST OF ACRONYMS AND ABBREVIATIONS

AF	Alternative Fuel
AFTM	Alternative Fuels Trade Model
AFV	Alternative Fuel Vehicle
BBL	Barrel, 42 U.S. gallons
BGE	Barrel Gasoline Equivalents
BGSE	Barrel-Gasoline Service-Equivalent
BTU	British Thermal Unit
CNG	Compressed Natural Gas
E85	A blended mixture of 85% ethanol and 15% gasoline
EIA	Energy Information Administration
EPACT	Energy Policy Act of 1992
FFV	Flexible Fuel Vehicle
FOE	Fuel Oil Equivalent
GAMS	General Algebraic Modeling System
GGE	Gasoline Gallon Equivalents
IRPE	Incremental Retail Price Equivalent
LPG	Liquefied Petroleum Gases
M85	A blended mixture of 85% methanol and 15% gasoline
MMBTU	Million British Thermal Units
MNL	Multinomal Logit
NPV	Net Present Value
MPG	Miles Per Gallon
MTPD	Metric Tons Per Day
NMNL	Nested Multinomal Logit
TAFV	Transitional Alternative Fuels Vehicle
USDOE	U.S. Department of Energy

1.0 INTRODUCTION TO THE TAFV MODEL

The Transitional Alternative Fuels Vehicle (TAFV) simulates the use and cost of alternative fuels and alternative fuel vehicles (AFVs) over the time frame of 1996 to 2010. As the model's name suggests, the TAFV model is designed to examine the *transitional* period of alternative fuel and vehicle use. That is, the model is a first attempt to characterize how the United States' use of AFVs might change from one based on new technologies available at a higher-cost and low-volume, to a world with more mature technologies offered at lower cost and wider scale. It also seeks to explore what would be necessary for this transition to happen, and what it would cost.

Previous studies of alternative fuels and vehicles differ in their estimates of the penetration rates and costs of AFVs. The Alternative Fuels Trade Model (AFTM, USDOE 1995, Leiby 1993) for example, found that there could be substantial penetration of alternative fuels and vehicles in 2010. Many of these studies are limited in that they examine AFVs in a single year. They present a 'snapshot' of AFV use given assumptions about technological maturity and price. The AFTM, notably, assumed mature vehicle technologies produced at large scale and a well a developed alternative fuel retail sector. Other studies, which examine AFVs in a multi-year, dynamic setting (e.g., Rubin 1994 and Fulton 1994) take technologies and prices as exogenously given. That is, they do not examine the important linkages between investments in alternative fuel and vehicle production plants, investment in vehicle stocks, investment in alternative fuel retailing infrastructure, and the prices and availability of fuels and vehicles.

This work follows up on the long-run equilibrium analysis done with the AFTM. The AFTM was developed to evaluate the long-run (2010) substitution of alternative fuels for gasoline, for a study pursuant to the Energy Policy Act of 1992 (Section 502b). The AFTM tracks supply, trade, and demand for multiple liquid and gaseous fuels in the interrelated energy markets of six world regions. It is a partial equilibrium model, used for long-run comparative static analyses. These analyses suggested that the prospective long-run substitution of alternative fuels for gasoline could be substantial, assuming that vehicles and fuels are widely available to consumers, and that the needed investments are made over time for the fuel and vehicle supply industries to gain cost savings from large scale production.

By making the scale of alternative vehicle and fuel production and the retail availability of alternative fuels endogenous, the TAFV model fills a gap in alternative fuel analysis. In contrast to the AFTM, the TAFV model specifically characterizes the time path of investment and adjustment, in order to consider whether some of these transitional issues may be important. The results from the TAFV model do, necessarily, reflect its many primary assumptions. Included are assumptions about the unit prices for vehicle and fuel production capital, the costs of raw materials, and input-output relationships which describe the productivity of a unit of capital in its respective employment.

This document summarizes the structure of the TAFV model, reviews the theory behind its equations, and documents many of its numerical inputs.

1.1 Principal Objectives of the Model

The principal objective of the TAFV Model is to provide a flexible, dynamic-simulation modeling tool that can be used for policy analysis. One use of the TAFV model is to assess possible ways in which early AFV mandates or incentives may influence the AFV transition. As alternative vehicle and fuel producers gain cumulative experience, some cost reductions through learning and economies of scale are expected. If vehicle manufacturers are encouraged to design and introduce new models with AF capability, the number of vehicle makes and models offering AF capability rises, and consumers value this greater choice. Incentives or programs leading to the earlier development of fuel distribution infrastructure can increase fuel availability. This can greatly lower the inconvenience cost associated with refueling, lowering the effective cost of alternative fuels. Promoting the introduction of AFVs may allow consumers to gain familiarity, reducing their uncertainty about fuel/vehicle performance and reliability. Programs calling for the purchase of AFVs by fleets lead eventually to the sale of used fleet vehicles to private consumers, making AFVs available to used-vehicle buyers, increasing consumer familiarity with AFVs and alternative fuels, and possibly leading to expanded private demand for alternative fuels and AFVs. Each of these possible linkages may work slowly, as investments are made and vehicle and capital stocks adjust.

1.2 Features

The TAFV Model is adaptable to many different policy scenarios. As currently configured, it:

- Solves yearly, 1996 - 2010;
- Is parameterized for the US urban and non-urban regions;
- Tracks the on-road vehicle stock by vehicle technology, fuel type and vintage;
- Tracks sales of new vehicles in each year by vehicle technology and fuel;
- Tracks installed capacity of methanol production;
- Tracks installed retail fuel capacity by fuel type;
- Tracks capacity utilization for vehicle production, fuel production, and fuel retailing;
- Accounts for the impacts of fleet mandates on manufacturer, consumer, and fuel retail behavior; and
- Estimates the costs and benefits of various policy scenarios.

1.3 Sectors Represented

The TAFV Model characterizes, in varying degrees of detail, interactions among the following major components or modules:

- Consumer and fleet vehicle demand and vehicle choice,
- Consumer and fleet fuel choice and use,
- Retail fuel supply and availability,
- Vehicle production,
- Motor fuel production,
- Raw material (retail fuel feedstock) supply.

These major interaction of these modules are depicted in ?.

1.4 Demand for Vehicle Transportation Services Drives Model

Consumer benefits are derived from the final demand for passenger vehicle transportation services. The final demand for transportation services is divided into three broad sectors: urban, non-urban, and fleet. These broad aggregates are further broken down into separate market segments. For each market segment, the model finds a point on the vehicle services demand curves where the marginal benefits of consumption (willingness to pay or price) equal the marginal costs of producing the needed composite mix of vehicles and fuels. In each period this balance is found subject to the limits of current vehicle and fuel production capacities, and the existing vehicle stocks. Investment in durable new vehicle and fuel production capacity and in new vehicles, is made based on a balance of the marginal investment cost with the expected lifetime value of the investment.

1.5 Principal Variables

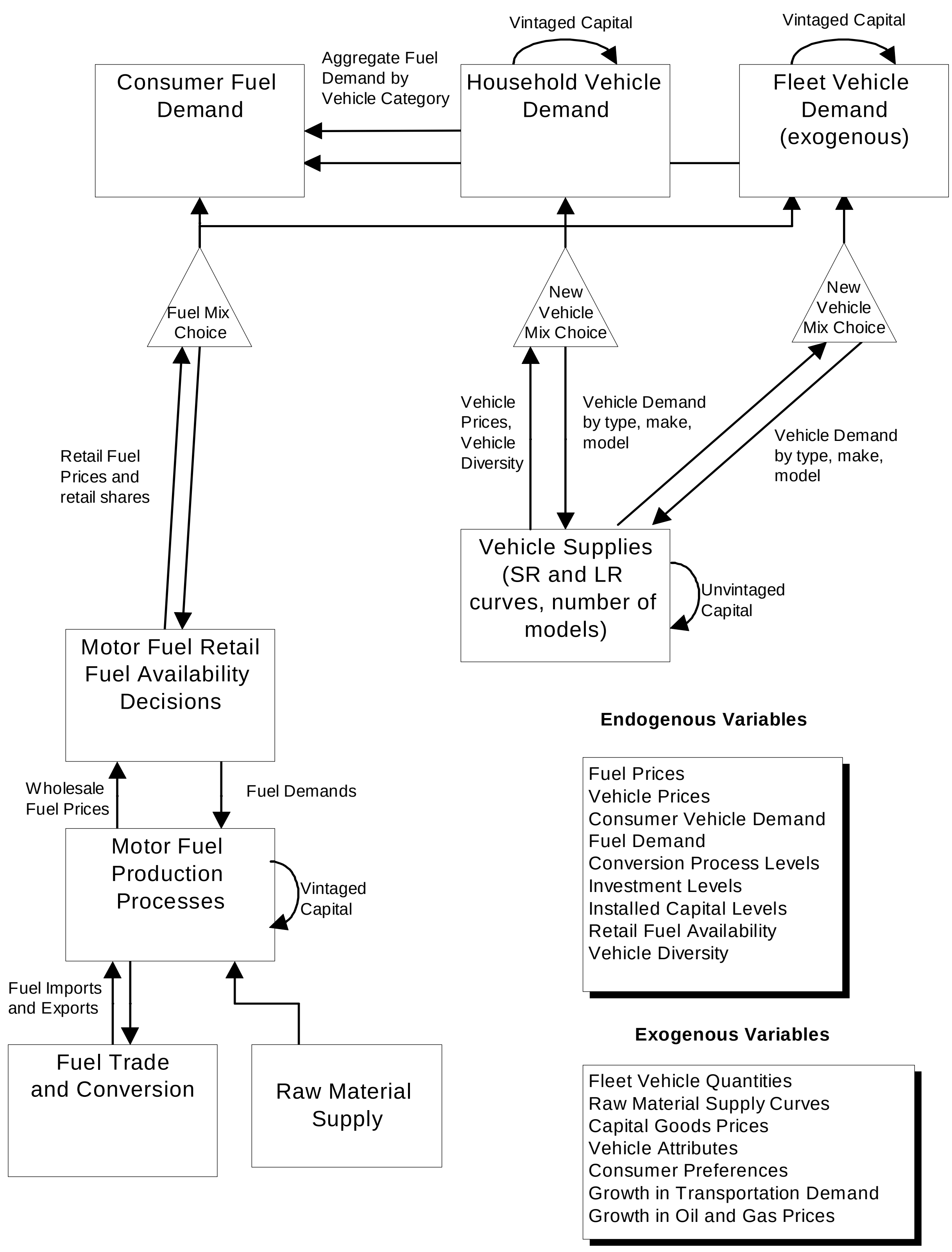
In each period *t* and region *r* the principal decision variables are:

- demand quantities for commodities d_{trf}
- supply quantities for commodities (fuels, vehicles) s_{trf}
- conversion process activity levels a_{trc}
- investments in new process capacity I_{trc}
- levels of installed capital for conversion processes K_{trc}
- fuel retail supply availability (share of stations) σ_{trf}
- vehicle supply diversity (makes & models) ρ_{tgv}

1.6 Cost Function Representation of Modules

This model take an economic approach, seeking to describe the results of many competitive agents (firms and consumers) each pursuing their own economic interests in a competitive

Figure 1: Diagram of TAFV Model



market. Therefore, rather than applying explicit behavioral relations and rules (e.g., assuming a particular rate or timing of vehicle introduction or fuel production and retailing), we seek to characterize the economic costs, profits, and consumption benefits of each activity. In this approach, each module *m* is represented in terms of its effective cost function C_{trf}^{m}, defined for each time period *t*, region *r*, and fuel *f*. Examples of component costs are: vehicle and fuel production costs, fuel retailing costs, raw material supply costs, and sharing or mix costs associated with vehicle and fuel choice. The cost functions summarize the way in which changing levels of activities, inputs, and outputs affect the costs for each module, and implicitly define the cost minimizing behavioral relations for those module variables. The model solution provides market levels of activity and the implied prices for all goods.

In cases where the module involves investments with long-lived (multiperiod) cost and benefits, the module cost function also includes the costs of current investments minus the (expected) future value function F_{t+1} for all remaining fixed investments K_{t+1}. Thus each module presents itself to the integrating framework/equations in terms of the net cost of current activities and investments:

$$C_{trf} \;=\; C_{trf}^{v}(s_{tr},d_{tr},a_{tr},K_t) \;+\; I_{tr}\cdot C_t^{K} \;-\; \frac{\delta_{t+1}}{\delta_t}F_{t+1}(K_{(t+1)r},s_{tr},d_{tr},a_{tr},\vec{P}_0,\vec{P}_1,\ldots,\vec{P}_{t-1}) \tag{1}$$

Here we are purposefully general about which supply, demand, conversion activity, or capital stock variables may determine a module's contemporaneous variable cost function C_{trf}^{v} or future value function F_{t+1}. Some regulations, for example those requiring a specific fleet vehicle mix at certain years in the future, may also influence the future value function.

1.7 Market Balancing Conditions

For each period, the objective is to represent a short-run market balancing which results from *competitive* behavior. This means that we wish to assure that the following short-run competitive conditions are met, unless activities are constrained:[2]

- i. the marginal (private) cost of producing each commodity equals its price;[3]
- ii. the marginal (private) benefit of each demand equals its price;
- iii. the marginal profitability of each intermediate conversion activity is zero (unless constrained, in which case short-run profits can be positive or negative); and,
- iv. the marginal current period value of investment equals the price of capital minus

[2]Each of these conditions corresponds to a first-order condition for single-period social surplus maximization used to determine the market solution.

[3]In the cases where the sector invests in long-lived capital, the marginal cost includes the shadow cost of any incremental capital required for production.

the discounted expected future value of the equipment from the next period.[4]

We require incremental investment to be positive. If new investment is zero, the profitability of existing capital is insufficient to motivate new investment, and the stated condition is not met. Disinvestment may be desired, but is not allowed.

We find market clearing supplies, demands, trade, and conversion process levels *s*, *d*, *x*, and *a*. That is, in maximizing consumption benefits minus production costs, the following balance equation must be met. The market solution is calculated with GAMS (Brooke, Kendrick and Meeraus 1992). This equation states that supplies, plus net output from conversion activities plus net trades between regions must be greater or equal to demand.

$$s_{trf} + \sum_c A_{fc} a_{tcr} + \sum_i (x_{tfri} - x_{tfri}) \geq d_{trf} \quad \forall\ f,r,t \tag{2}$$

where:

r, i	index regions,
f	indexes commodities (fuels, vehicles),
c	indexes conversion processes,
t	indexes time,
d_{trf}, s_{trf}	demand and supply quantities,
a_{trc}	activity level for conversion process c,
A_{fc}	coefficient indicating commodity f output (input) per unit process c activity, and,
$x_{tf\rho r}$	shipment of commodity f from region ρ to r.

Final demands and basic commodity supplies are "price responsive" in that their quantities will depend on market prices in each period:

$$s_{trf} = S_{trf}(P_{trf}), \qquad d_{trf} = D_{trf}(P_{trf}) \ . \tag{3}$$

Fuel blending and conversion, fuel distribution and retail markup, and the combination of fuels with vehicles to provide vehicle services are represented with linear conversion processes. For conversion processes requiring durable capital equipment (such as methanol fuel production or vehicle production), the amount of installed capital imposes a constraint on the maximum activity level for the associated conversion process. In addition, there is a capital stock evolution constraint that links depreciated capital and investment in each period to the next period starting capital stock.

[4]We require incremental investment to be positive. If new investment is zero, the profitability of existing capital is insufficient to motivate new investment, and the stated condition is not met. Disinvestment may be desired, but is not allowed.

2.0 VEHICLE SERVICES DEMAND FOR NEW AND USED VEHICLES

Benefits in this model come from the satisfaction of final demand for transportation services. Total demand for transportation services is specified with a composite demand curve in each region/market. Composite transportation services is measured in Barrels of Gasoline Service Equivalents (BGSE), and the level of composite demand is benchmarked to the EIA projections of motor fuel demand. In each period this composite demand may be satisfied by the use of existing (used) vehicles and the purchase and use of new vehicles. The use of older vehicles is limited by the stock of each type. The capital cost of used vehicles is treated as sunk, and the allowable use of each vehicle is fixed by age.

A mix of new vehicles is purchased to the extent that existing vehicle stocks are insufficient. New vehicles are chosen according to the Nested Multinomal Logit (NMNL) choice formulation (Greene, 1995; Leiby and Greene, 1995) based on vehicle capital costs, non-price attributes, vehicle model diversity, and *expected* lifetime nested fuel choice costs. In this way, long-lived investment consequences are reflected in vehicle choice. Note that under myopic expectations, lifetime fuel costs follow from current fuel costs. Fuel choices must be made for the vehicles which are dual or multi-fueled.

The level of use of each vehicle type (in miles traveled) is assumed fixed, but declines with vehicle age. Since the capital costs of existing vehicles is sunk, there is a strong expectation that transportation demand will be satisfied by existing vehicles before new (or used fleet) vehicles are purchased. The existing vehicle stock provides an upper limit on the amount of vehicle services that existing vehicles may provide. Older, existing vehicles *may* be underutilized if they are so unappealing that the purchase of a new vehicle is a lower cost alternative.

As demand shifts out over time and older vehicles are scrapped, the household sector must acquire more vehicles. There are two possible approaches to modeling incremental vehicle choice:

> (i) Choices of the incremental vehicle mix are independent of the mix in existing stock. They depend on current new vehicle attributes and current consumer preferences, and the consumers purchasing new vehicles are assumed to be a random sample of all consumers.
>
> (ii) Choices of incremental vehicle mix are determined by applying the vehicle choice function to the entire fleet, with the mix of existing vehicles influencing the choice of new vehicles. The entire fleet of old and new vehicles used to satisfy aggregate transportation services demand is evaluated based on *current* consumer preferences. This approach is closer to assuming that consumers purchasing new vehicles are those who owned the retired vehicles, since there is more tendency for new vehicle purchases to be similar to the retired vehicles.

The model adopts the first approach. This means that the multinomial choice framework determines the shares of AFVs among the *new* vehicles purchased. If the vehicle prices, fuel prices, and non-price attributes were to remain constant, over many years of new vehicle purchasing and used vehicle scrappage, the AFV shares in the entire vehicle stock would tend toward the shares in new vehicle purchases. Of course, vehicle and fuel prices are expected to vary, as are some endogenous non-price attributes, such as retail fuel availability and AFV model diversity. So the market may continue in transition so long as the new vehicle fuel technology mix differs from the mix in the existing vehicle stock.

2.1 The Choice of Vehicle Mix to Satisfy Vehicle Demand

Some demands may be satisfied by a mix of alternatives, where the mix is sensitive to relative prices and non-price attributes. In the TAFV model, examples are the demands for *new* vehicles, and for fuels by multi-fuel vehicles. Those demands may be represented by a composite good, q_{trg}, with the requirements:

a. That the level of "production" of the composite good, g, equals the sum of the levels for its alternative inputs, which are designated by the set F_g.

b. That the shares of alternative inputs f for composite good v conform to the expected sensitivity to relative price P_f and non-price attributes α_f .

Note that price and non-price attributes may vary by time and region (as could the attribute sensitivity parameter β). Consumers' demand for vehicles is driven by a price-responsive demand curve for vehicle services, which will be satisfied by an endogenously determined mix and quantity of vehicles and fuels.

Some attributes of flexible choice for new vehicles in particular may depend on both historical experience and future expectations. For composite vehicle services demand of type g (q_{trg}) the choice among input alternatives, f, will depend upon their (conditional expected indirect) utility, V_{trgf}, which is a linear function of new vehicle price P_{trf} and non-price attributes:

$$V_{trgf} = \beta_g(P_{trf}+\alpha_{fR}+\alpha_{fW}+\alpha_{fD}+\frac{\beta_f}{\beta_g}C_{gf}) \tag{4}$$

The attributes include, for example:

- β_g — cost sensitivity parameter for choice over vehicle types
- P_{trf} — vehicle price for fuel technology f, at time t in region r;
- α_{fR} — vehicle range (distance between refuelings) equivalent cost;
- α_{fW} — vehicle weight/performance equivalent cost;
- α_{fD} — relative diversity of vehicle models, equivalent cost;
- β_f — fuel price sensitivity for vehicle f
- C_{gf} — *expected* effective fuel cost over vehicles lifetime, given current and expected future prices for the fuels vehicle f may use, and accounting for expected fuel

availability.

Thus endogenous current conditions (prices and fuel market share) affect at least two choice parameters, and expectations about future prices and fuel availability must also be considered. It is useful to bear in mind the following points.

The resale value of a vehicle will depend not only on its age, but also on its current market attractiveness, which depends in turn the price and availability of new substitutes, and on expected future fuel prices. Hence the resale value of a vehicle is endogenous and path dependent.[5]

Fuel availability is represented by the share of service stations offering the fuel, and is an output of the retail sector module.

Vehicle prices are determined by vehicle production sector decisions, as well as expected resale or scrappage values in cases where foresight of the decision-making time horizon is limited.

2.2 Factors Influencing Vehicle and Fuel Choice Shares

Vehicle and fuel choice behavior in the TAFV model depends upon the prices and the non-price attributes of each alternative. Greene (1994) explicitly stipulates a set of key attributes, quantifying their differences across vehicles and fuels, and establishing estimates of how consumers may value them. It is also necessary to estimate the sensitivity of consumer choices to differences in prices or effective costs (denominated in dollars).

The treatment of vehicle and fuel choice in the TAFV model is based on Greene's "Alternative Vehicle and Fuel Choice Model," (AFVC, Greene 1994). It is always problematic to estimate the demand for novel vehicle or fuels, since there is little direct historical experience from which to draw inferences. Due to the limitations and potential biases of stated preference surveys, Greene's strategy was to "draw inferences, based on revealed preferences expressed in analogous but different situations, about how consumers value the attributes of similar goods." (Greene 1994:5) Examples of such analogous situations include choices among different fuel grades, choices between gasoline and diesel engines, and choices among different conventional automobile types.

The resulting choice shares for given prices and attributes apply to new vehicle purchases. The choice will also influence the long-run vehicle stock composition through the process of new vehicle purchase and vehicle aging and retirement.

The price slope coefficients (which also apply to the effective cost of non-price attributes) are

[5]In the TAFV model approach this estimation of used car values is automatically handled through the calculation of the shadow price of the used vehicle utilization constraint.

calculated from a stipulated elasticity of share with respect to price, specified at a given market share and vehicle/fuel price.[6]

The choices among vehicle and fuels are established in a nested-multinomial logit (NMNL) structure. The conditional choice of fuel within a subset of alternatives given a particular vehicle type is assumed to be of the Multinomial Logit (NML) form. The structure of this nesting indicates naturally comparable choices. However, the principal reason for segregating vehicle service cost components into fuel and vehicle-specific categories is to allow the use of different cost sensitivity coefficients. Deeper nests correspond to conditional choice (such as that for fuels given a vehicle), and are expected to be more price sensitive. For the NMNL to be properly structured, the choices within a nested subset must be closer substitutes than choices at a higher level.

2.2.4 Effect of Motor Fuel Costs and Fuel Mix Choice on New Vehicle Choice

At the point of new vehicle choice, the consumers choose among alternatives based on the effective cost of *vehicle services,* measured in dollars per barrel of gasoline equivalent services. This service price includes the cost of fuel and the marginal capital cost of the vehicle amortized over barrels of fuel use. The effective cost of fuel and vehicle non-price attributes (such as fuel availability and limited vehicle diversity) are also reflected in the cost of vehicle services. For multi-fueled vehicles, the nesting of fuel choice within the vehicle choice problem is handled by passing composite fuel prices (including fuel choice sharing costs) to the vehicle choice function.

Fuel choice depends on fuel attributes such as price, vehicle performance using the fuel, and refueling convenience. It also depends upon current fuel availability, that is the fraction of retail stations offering the fuel. This variable is endogenously determined in the fuel retail sector. The multinomial fuel choice function uses an indirect utility for each fuel which is linear in fuel price and includes a constant term to reflect most other attributes. The indirect utility (or effective cost) for each fuel varies non-linearly with endogenous fuel availability. The effective cost of fuel availability is expected to be quite high for availabilities below a few percent, and to decline to near zero as availability exceeds some moderate level (currently on the order of twenty percent). The cost of low retail fuel availability is an important factor in the transitional analysis. It depends on the additional travel and inconvenience which is required to refuel when stations are rarer, and on the consumer's valuation of that additional travel. Further discussion of the effective cost of limited retail fuel availability may be found in Chapter 6.

[6]The slope Beta is both the price/cost sensitivity parameter in the utility function and a scale parameter inversely related to the standard deviation of the error term in the underlying random utility choice model. For larger Beta, choices are more price sensitive, indicating that the choices are close substitutes and that the random utility error term is smaller. 1/Beta may be interpreted as the marginal utility of dollars.

The endogenous and exogenous factors influence the choice among vehicles and fuels in the TAFV are given in the table 1 below.

Table 1: Factors Influencing Vehicle Choice

Factors considered in Fuel Choice	Endogenous/Exogenous
Fuel Price	Endogenous
Fuel Availability (fraction of retail stations offering fuel)	Endogenous
Refueling Frequency (based on range)	Exogenous
Refueling Time Cost	Exogenous
Performance Using Fuel (horsepower-to-weight ratio changes)	Exogenous
Factors Considered in Vehicle Choice	**Endogenous/Exogenous**
Vehicle Price	Endogenous
Fuel Cost (including the effective cost of non-price fuel attributes, as expected average of fuel mix for multifuel vehicles)	Endogenous
Performance (measured by changes in the horsepower-to-weight ratio)	Exogenous
Cargo Space (based on loss due to space required for fuel storage)	Exogenous
Vehicle Diversity (number of models offering AFV technology)[7]	Endogenous

In the Greene (1994) AFVC analysis, the multi-fuel capability of some vehicles was also ascribed a separate "option" value, since it would allow drivers to change fuels as fuel price differentials fluctuate. No separate value is ascribed to the multifuel attribute in the TAFV model, for two reasons: the option value is difficult to calculate given endogenously varying fuel prices and availability; and since some of the fuel flexibility option value is automatically included when fuel prices change over the time horizon of the model. Alternative fuel vehicles are assumed to be equally reliable and safe as conventional vehicles. Any social or aesthetic benefits of alternative fuels or vehicles are omitted from the private choice determination.

[7]There is an extended discussion of this issue later in this chapter.

Amortization of Vehicle Capital Costs Into Capital Charges

In the Greene (1994) AFVC model analysis, vehicle costs are amortized to an equivalent capital charge per unit vehicle services with capital recovery factor of 16.8%/year, and allocated over an estimated 534 gallons of fuel use per year (12.72 BGE/year). This capital recovery factor is calculated based on Greene's assumed discounting (7%) and vehicle depreciation (15%) rates. In contrast, in the TAFV model, capital costs are borne in a lump sum at vehicle purchase time, and fuel charges are borne as the vehicle is used over time. However, the capital charge rate implied by the TAFV dynamic cost minimization outcome closely matches that used in the Greene long-run static analysis. The TAFV implied capital charge rate of roughly 17% can be calculated from the method described in Appendix 2, based on the TAFV model's 10% discount rate, historically-based vintaged scrappage rates, and a 5.4% rate of declining vehicle use with age.

2.3 Effective Cost to Consumers of Limited Vehicle Model Diversity

Consumers care about many vehicle attributes other than those specifically related to fuel technology. If the alternative fuel technology is offered on only a limited number of vehicle models, consumers are less likely to find the mix of attributes they prefer most. Accordingly, consumers will view those vehicle fuel technology types with comparatively limited model diversity as having an added effective cost. In TAFV, model diversity is measured relative to conventional gasoline vehicles. By limited relative diversity we mean that the alternative fuel technology f is offered on fewer models N_f than the reference alternative of a conventional gasoline vehicle, with N_0 models. The demand for a particular AFV technology will increase with the diversity of vehicle classes and models for which it is offered. To capture the cost of limited diversity we adopt and extend the specification proposed by Greene (1995). In this approach, for vehicles of each fuel type f, a limited-diversity cost term C_f^V is included in the generalized vehicle cost to account for the costs of limited model diversity. Rather than explicitly modeling the different vehicle models and the consumer choice among models, this specification provides a powerful simplification which lets us account for diversity solely in terms of the number of vehicle models offered for each fuel technology type.

Within the nested multinomial choice framework the random utility function for individual i of choice f is given as: [8]

$$U_{if} = V_f + U_f^D(n_f) + \varepsilon_{if} \tag{5}$$

where V_f is the conditional indirect utility of the usual priced and non-priced attributes, U_f^D is the utility limited vehicle diversity n_f, and ε_{if} is a random term reflecting the contribution of unobserved attributes for vehicle technology type f specific to individual i. Letting P_f be the price of the measured attributes and β be the marginal utility of a dollar change in vehicle price, then the utility of the measured attributes other than model diversity can be represented as a linear function of the prices of the attributes.

[8]See Maddala, Chapter 3 for an excellent treatment of nested multinomial logits.

$$V_f(\vec{P}) \;=\; \alpha_f + \beta P_f \tag{6}$$

2.3.1 Theoretical Valuation of Vehicle Diversity: Random Model Introduction Case

The utility of model diversity stems from the benefit consumers gain from having a choice over a variety of vehicle models, within each fuel technology type *f*. In the TAFV Model, this selection among models corresponds to a *nested* multinomial logit choice. Greene (1995) shows that if all vehicle models within a fuel technology type (and size class) are essentially equally attractive, then the utility of vehicle model diversity can be represented as

$$U_f^D \;=\; \ln\left(\frac{N_f}{N_0}\right) \;\leq 0 \quad \forall\; N_f \;\leq\; N_0 \tag{7}$$

where N_f is the number of models of fuel type *f* and N_0 is the number of models of a numeraire vehicle, e.g., a conventional gasoline vehicle. The numeraire vehicle is presumed to have the widest diversity, i.e., $N_f \leq N_0$. Note that the loss from limited diversity is zero if $N_f = N_0$. For model diversity less than n_0, the utility is lower (negative).

The overall utility of a vehicle is then given as the sum of its measured and non-measured attributes.

$$U_f(P_f, N_f) \;=\; \alpha_f \;+\; \beta P_f \;+\; \ln(N_f/N_0) \tag{8}$$

For the numeraire vehicle, the utility is simply

$$U_0 \;=\; \alpha_0 \;+\; \beta P_0 \tag{9}$$

This commonly used indirect utility function which is linear in price implies a constant marginal utility of income. Accordingly, we can divide by the marginal utility of dollars (-β) to infer that the monetary cost equivalent of limited diversity is $\ln(N_f/N_0)/\beta$, in dollars per vehicle. The vehicle choice calculation (based on the sharing-cost equation) includes this monetary cost of limited vehicle diversity. The total consumer cost of limited number of models N_f, for the consumption of $\boldsymbol{q_f}$ vehicles of fuel type *f*, is

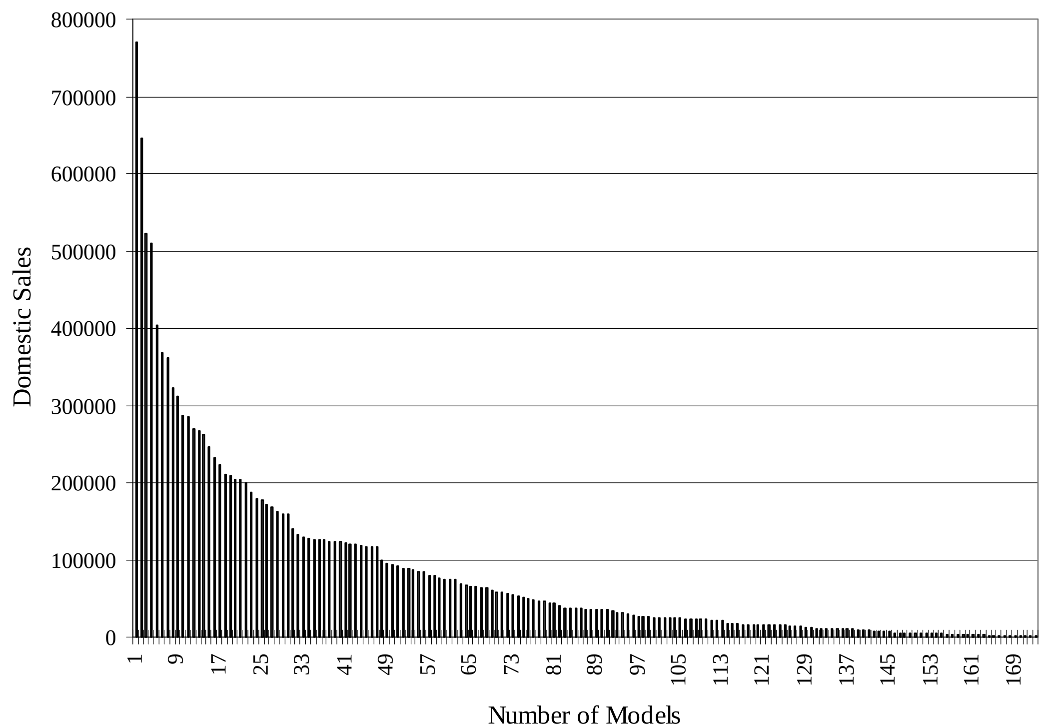
$$C_f^V \;=\; \frac{1}{\beta}\ln\left(\frac{N_f}{N_0}\right) q_f \tag{10}$$

where β is the NMNL choice parameter that applies to the choice among vehicles types. The units make sense in this diversity cost function, since β is denoted in utils per \$/vehicle, $\ln(N_f/N_0)$ is a unitless number of "utils," quantity q_f is measured in new vehicles, so cost C_f^V will be in dollars.

2.3.2 Vehicle Model Popularity and the Order of Alternative Fuel Technology Introduction

The above formulation is based on the implied utility from a nested multinomial logit choice over vehicle models, within each fuel technology class. It presumes that while each consumer may view the available models that offer the fuel technology somewhat differently, overall the models are essentially equally popular. We know, of course, that within a particular size that the popularity (market share) of conventional vehicles varies widely. In fact, we observe that the vehicle production shares vary widely across models (see Figure 2). Hence, an assumption of equal model popularity would not be good. An alternative interpretation is that the above diversity cost measure assumes that the alternative fuel technology is introduced on vehicle models in a random order. This means that if the fuel technology is offered on n_f out of n_0 possible models, on average a fraction of N_f/N_0 consumers could obtain the technology on their favorite model. We can roughly say that a N_f/N_0 share of consumers' demand for models will be satisfied. A random order of alternative technology introduction on a set of unequally popular models implies the same utility of diversity as any order of introduction on a set of essentially equally popular models.

Figure 2: Domestic Vehicle Sales by Model
(All Light Duty Vehicle Classes)



There is some reason to believe that vehicle producers may offer alternative fuel technologies on the most popular vehicle models first. This could increase the chances of the AFV's success, and be profitable for firms. The nested MNL choice model implies that the utility of diversity is logarithmic in the share of consumers' demand satisfied by the number of vehicles offered. We can generalize our valuation of vehicle diversity by writing the utility of diversity as a function of share of demand satisfied, S(n).

$$C_f^V = \frac{1}{\beta}\ln\bigl(S(N_f)\bigr)q_f \tag{11}$$

The treatment above implies that the (average) share satisfied by each model introduced was $1/N_0$. In this case the number of models offered N_f the share satisfied is:

$$S(N_f) = \frac{N_f}{N_0} \tag{12}$$

Suppose instead that, within each vehicle type, each of the models has popularity share s_n. The total popularity share for N_f models offered is then

$$S(N_f) = \sum_{N=1}^{N_f} s_N \tag{13}$$

Looking at Figure 2 which is based on actual 1994 vehicle production data, we see that if AFV models are introduced in order of their popularity, the incremental share satisfied is a reasonably smooth and well-behaved function, which declines in N_f. This *cumulative* share satisfied curve can be roughly matched by a power function of N_f, i.e.:

$$\begin{aligned} S(N_f) &\approx \left(\frac{N_f}{N_0}\right)^{\omega} \\ S(0) &= 0 \\ S(N_0) &= 1 \\ \omega &> 0 \end{aligned} \tag{14}$$

Substituting this form into the diversity cost function:

$$\begin{aligned} C_f^V(q_f,N_f) &= \frac{1}{\beta}\ln\left(\frac{N_f}{N_0}\right)^{\omega} q_f \\ &= \frac{\omega}{\beta}\ln\left(\frac{N_f}{N_0}\right) q_f \end{aligned} \tag{15}$$

The useful result is that we can account for the different popularity of models and alternative orders of AFV model introduction with a single scaling parameter ω. If AFV technology is introduced on models in order of their popularity, then the incremental popularity share satisfied by an additional model declines with the number of models. In this case the cumulative share function $S(N)$ is a concave function of N ($0 < \omega < 1$). For AFV technology introduction which is random in terms of model popularity, cumulative share $S(N)$ is linear function of N (i.e., $\omega = 1$). Interestingly, we see that if AFV models are offered in order of their popularity rather than randomly, the implied costs of limited diversity are *diminished*, since the estimated parameter is in the range between zero and one. The intuitive explanation for this result is that when a small number of models are offered the popularity-based introduction means that more consumers are satisfied and there is less initial concern about model diversity. Also, in this case the marginal contribution to diversity from adding another new model declines.

Chapter 3 describes the benchmarking of initial model diversity N_f and the order of model introduction scaling parameter ω from current vehicle model sales data.

2.3.3 Marginal and Unit Costs of Limited Diversity

The marginal consumer cost of increasing the number of models (measured in $/model) is:

$$\frac{\partial C_f^D(q_f,N_f)}{\partial N_f} = \frac{\omega}{\beta_v}\frac{q_f}{N_f} < 0 \tag{16}$$

As expected, the marginal change in the cost of limited diversity for increasing diversity is less than zero (the cost of limited diversity declines with N_f), since β_v is negative. Increasing the number of makes or models is beneficial, and lowers the cost to consumers from having a limited choice of vehicles. Stated differently, we can refer to the negative of the marginal cost of limited diversity as the marginal benefit of increased model diversity. Dividing by the number of vehicles q_f, the marginal *benefit* of increased model diversity per vehicle is positive and declines inversely with N_f:

$$-\frac{1}{q_f}\frac{\partial C_f^D(q_f,N_f)}{\partial N_f} = \frac{\omega}{-\beta_v}\frac{1}{N_f} > 0 \tag{17}$$

Its value, measured in ($/vehicle)/model, depends only on diversity N_f, the marginal utility per dollar β_v and the order-of-introduction scaling parameter ω. For any given level of vehicle production q_f, the market equilibrium level of model diversity will balance the marginal benefit of increased diversity N_f to consumers with the marginal cost to vehicle producers of expanding model diversity.

Also, the effect of limited diversity on the marginal consumer cost per vehicle is constant for a given diversity N_f

$$\frac{\partial C^D}{\partial q_f} = \frac{\omega}{\beta_v}\ln\left(\frac{N_f}{N_0}\right) \equiv \underline{C}^D(N_f) < 0. \tag{18}$$

Accordingly, we call this the "unit cost of limited vehicle diversity," that is the added marginal cost per vehicle due to limited diversity of vehicle models. Its value, measured in dollars per vehicle, depends only on *relative* diversity N_f/N_0, the marginal utility per dollar β_v and the order-of-introduction scaling parameter ω. This unit cost of limited diversity is positive, since both β_v and $\ln(N_f/N_0)$ are negative (for $N_f<N_0$). For any given level of model diversity N_f, the market equilibrium level of vehicle sales q_f will balance the marginal benefit of vehicles with their marginal production costs plus their effective non-price attribute costs, including the unit cost of limited vehicle diversity.

By a second differentiation of (18) or (16) we get the following expression

$$\frac{\partial^2 C^D}{\partial N_f \partial q_f} = \frac{\omega}{\beta_v} N_f < 0 \tag{19}$$

which represents the rate of change (decline) in the marginal costs of diversity with the number of vehicles purchased. This cross-partial derivative is independent of the number of vehicles q_f.

Since vehicle choice in TAFV is done in terms of new vehicle services, the vehicle services MNL coefficient β_s does not have the correct units for the above cost function. The units of β_s are utils/($/BGSE), so we need to adjust them for the number of vehicles required per barrel-gasoline service-equivalent (BGSE) provided. The appropriate conversion factor is the stock-per-flow coefficient κ for vehicle stock providing vehicle services. The units of κ are million vehicles per billion BGSE, i.e. vehicles per 1000 BGSE. Thus we can use $\beta_v = \kappa\beta_s$, which has the units of utils/($1000/vehicle). Accordingly, the unit cost of limited diversity per vehicle will be measured in $1000/vehicle, and, for q_f in million vehicles per year, the diversity cost will be in $billion per year.

2.3.4 Vehicle Model Diversity Data and Benchmarking Costs of Limited Model Diversity

To benchmark diversity value (and the effective cost of limited diversity), we must set the numeraire vehicle diversity N_0, and order-of-introduction parameter ω. The numeraire vehicle diversity is identified with the diversity of conventional gasoline vehicles, which depends on what we define as a distinct model. The definition of a model from the perspective of consumers must match what is meant by a model from the perspective of the vehicle production module. In the vehicle production module the addition of each new model is treated as requiring a new plant (or comparable investment. That is, it imposes added costs equivalent to adding a new line or production plant, with the scale of production equal to total industry capacity K divided by n. To benchmark the reference vehicle diversity, the number of distinct vehicles (each usually associated with a separate production plant) was counted from data in the *Market Data Book*, controlling for comparable vehicle models (e.g. Taurus and Sable) and omitting low production runs (<1000/year). The number of model types available for 1994 for automobiles was 128 and for light trucks 45. The Table below shows the breakdown of vehicle types and the classifications devised to produce the reference model diversity for conventional vehicles.

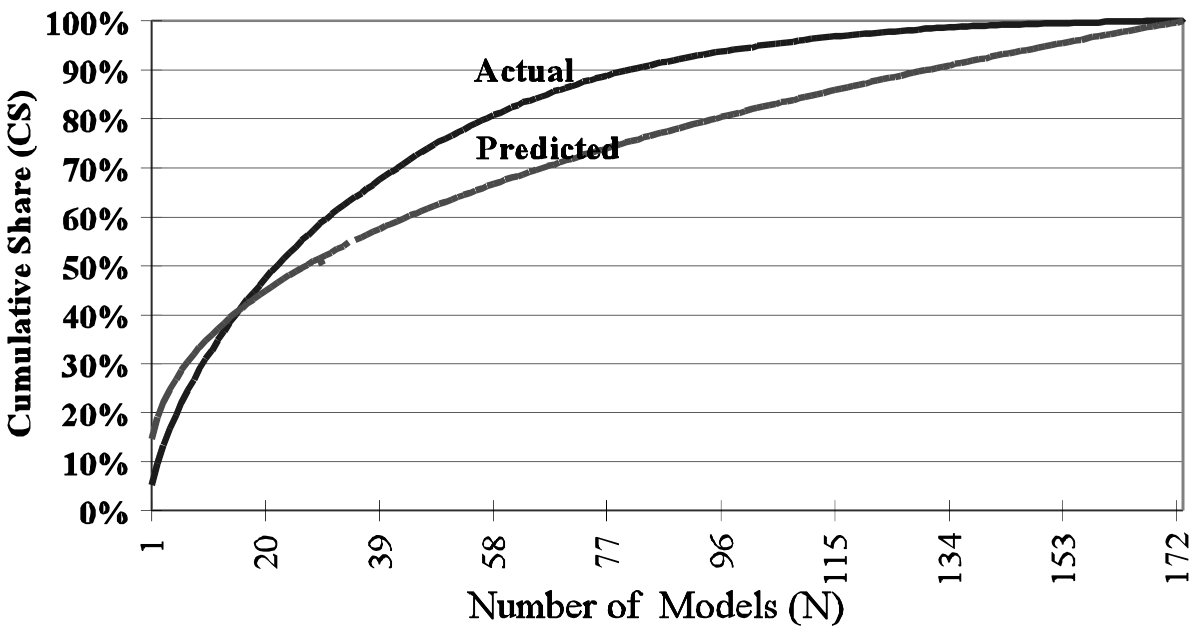
Table 2: 1994 Conventional Gasoline Vehicle Model Diversity, by Size Class				
Automobiles	**Number of Models (Diversity)**	**Low Sales Volume**	**Duplicated Models***	**Vehicle Type Total**
Two Seater	8	10	0	18
Minicompact	7	5	0	12
Subcompact	34	5	3	42
Compact	35	9	5	49
Small Total	84	29	8	121
Midsize	25	5	5	35
Large size	19	2	3	24
Large Total	44	7	8	59
Automobile Total	212	65	24	301
Cargo Truck	17	0	6	23
Passenger Truck	28	1	14	43
Light Trucks Total	45	1	20	66
Grand Total	173	37	36	246

For each of the vehicle size classes, the distribution of vehicle production volumes by model was used to estimate the curvature ω of the model cumulative popularity function, $S(n) \approx (n/n_f)^{\omega}$. For each size class these popularity curves were similar, and the curvature ω values were found to be close to 0.386.

Table 3: Estimate of Cumulative Popularity Share Parameter ω (Based on 1994 Model Sales Data)	
Small Automobiles	0.39
Large Automobiles	0.40
Cargo Trucks	0.38
Passenger Trucks	0.37
All LDV Classes	0.37

The fit is not excellent, but it provides a reasonable and simple approximation of the vehicle diversity and vehicle popularity issue. Using this value for ω, the predicted versus actual cumulative share of vehicle introduction for the combined data is shown in the figure below.

Figure 3 Fitted and Actual Vehicle Cumulative Market Share versus Number of Models



2.3.5 Data: Unit Costs of Limited Model Diversity per Vehicle - Numerical Examples

Given the coefficient β_s from the vehicles services MNL choice function of -0.0727 utils per dollar

BGSE, we find β_v = -2.475 utils/($1000/vehicle). Assuming that AFVs are introduced on models in order of model popularity, the limited-diversity cost scaling parameter ω is 0.37. For a numeraire number of models n_0 of 173, the unit diversity cost per new vehicle ranges from $770 (when only one model is offered) to $0 (full diversity). Under the less likely assumption of an essentially random sequence of model introduction, the cost of limited diversity ranges from $2080 to $0 per vehicle.

2.4 Data: Vehicle and Fuel Choice Parameters

The following tables show TAFV Model vehicle and fuel choice data. The first table provides specific numerical values for the non-price attributes of vehicles and fuels in the TAFV model. The second table reports vehicle choice shares and fuel choice shares for multi-fuel vehicles, for the special case where the endogenous choice variable (fuel price and availability, vehicle price and diversity) are equal. It provides some indication of the effect of the exogenous non-price attributes.

Table 5: Vehicle and Fuel-Specific Attribute Values in TAFV								
Vehicle	Fuel	Relative Energy Efficiency	Change In Vehicle Weight	Change in H.P.	Storage Space (gallons)	Storage in Units (gal, scf, kWh)	Refueling Time (minutes)	Additional Vehicle Cost ($/Vehicle)
Conventional	Gasoline	100.0%	0.00%	0.00%	15.8	15.4	6.0	$15,000
Flex-Fuel	Gasoline	100.0%	0.00%	0.00%	18.5	18.0	6.0	$15,243
Flex-Fuel	M85	101.0%	0.00%	3.00%	18.5	18.0	6.0	$15,243
Flex-Fuel	E85	101.0%	0.00%	3.00%	18.5	18.0	6.0	$15,243
CNG Bifuel	Gasoline	97.0%	4.70%	0.00%	51.8	15.4	6.0	$16,596
CNG Bifuel	CNG	97.0%	4.70%	-10.00%	51.8	650.0	7.5	$16,596
LPG Bifuel	Gasoline	98.0%	3.70%	0.00%	37.0	15.4	6.0	$15,813
LPG Bifuel	LPG	98.0%	3.70%	-5.00%	37.0	20.0	6.5	$15,813
CNG Dedicated	CNG	100.0%	6.50%	0.00%	49.9	1300.0	8.0	$16,451
LPG Dedicated	LPG	100.0%	2.00%	0.00%	21.2	20.0	6.5	$16,665
Alcohol Dedicated	M85	105.0%	0.00%	10.00%	18.5	18.0	6.5	$15,176
Alcohol Dedicated	E85	105.0%	0.00%	10.00%	18.5	18.0	6.5	$15,176
Electric Battery	EV	429.4%	0.00%	0.00%	38.7	53.5	360.0	$22,608

Table 6: Equal Fuel and Vehicle Price Market Choice Shares			
Vehicle	**Fuels**	**Fuel Share**	**Vehicle Share**
Conventional	Conventional Gasoline		16.9%
Flex-Fuel	Conventional Gasoline	19.0%	
Flex-Fuel	M85	40.20%	
Flex-Fuel	E85	40.20%	16.8%
CNG Bifuel	Conventional Gasoline	90.8%	
CNG Bifuel	CNG	9.2%	7.1%
LPG Bifuel	Conventional Gasoline	76.0%	
LPG Bifuel	LPG	24.0%	13.8%
CNG Dedicated	CNG		9.7%
LPG Dedicated	LPG		15.6%
Alcohol Dedicated.	M85	50.0%	
Alcohol Dedicated	E85	50.0%	19.4%
Electric	Battery EV	0.0%	0.6%
Total Vehicle			100.0%

3.0 VEHICLE STOCK, USE AND DATA

3.1 Vehicle Stock Equations

The stock of vehicles is tracked by age and year of manufacture (vintage). This allows vehicles to have characteristics change over time. An exogenously determined scrappage profile is applied to vehicles by age group. In addition, vehicle use (gallons of fuel consumed per year) declines with the age of the vehicle. This allows us to determine, for example, how many four-year-old dedicated CNG vehicles are on predicted to be on the road in the year 2000 and how much fuel they use.

One complicating factor to consider with *flexible* fuel vehicles is that prior to purchase consumers have a choice of the type of vehicle (e.g., CNG, alcohol, LPG) and whether to use gasoline or the alternative fuel. Once the vehicle is purchased, consumers can only choose over which fuel to use. Since vehicles and fuels are chosen in a multinomial choice framework based upon their attributes (see Section 2) newly purchased vehicles, that provide vehicle services in the year purchased, need to be differentiated from the on-road stock of vehicles to allow for this bifurcated vehicle-fuel, fuel-only choice. With dedicated fuel vehicles this complication obviously does not exist. To keep the structure of the model consistent, however, we treat all vehicle types identically. In the vehicle stock equations of motion shown below ?, therefore, new investment in vehicles by year, region and type, I_{trc}, is treated as an age 0 vehicle, K_{tr0c}, and tracked separately from vehicles of age 1 and beyond.

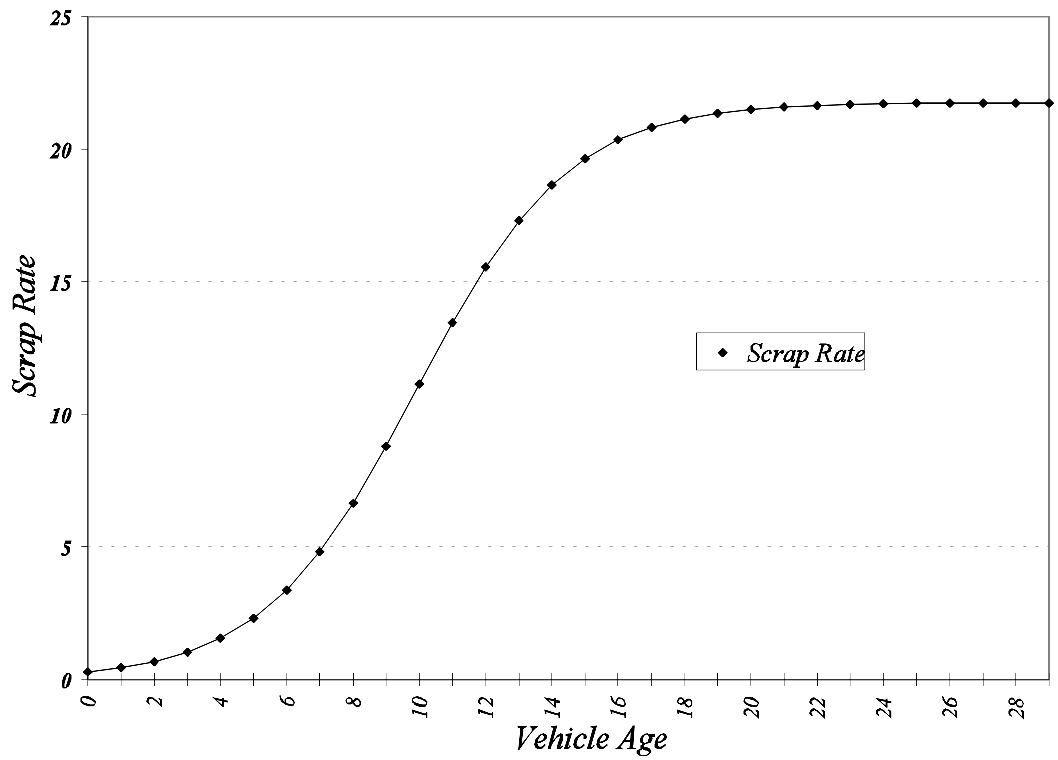
Vehicle scrappage and use rates are combined into a single factor, γ_{ac}, which varies by age. Since these factors are exogenous they are independent of new vehicle choice and applied to brand new vehicles. This makes that assumption that new vehicles survive to be one year old. For vehicle type *c* the vehicle equations are given below.

$$\begin{aligned} K_{(t+1)r(a+1)c} &= K_{trac}(1-\gamma_{ac}) \quad \forall\ t < T, r, a, c \\ K_{tr0c} &= I_{trc} \end{aligned} \tag{20}$$

3.2 Vehicles Stock Data

Since the TAFV model explicitly tracks vehicles by fuel type, year of manufacture (vintage), and age, it is necessary to know the age distribution by fuel type in the models initial year. Moreover, the rate at which vehicles are scrapped and the amount they are driven also depend on their age. The surviving stocks of gasoline vehicles by vintage for the years 1984 - 1994 are derived by combining data for automobiles and light-duty trucks (0-14,000 lbs) (AAMA, pp. 39-40, Davis and McFarlin, pp. 3-20). In the TAFV model, vehicles that are not scrapped, can live up to 30 years. Therefore, we estimated the surviving stocks of on-road vehicles for vintages 1965-1983

Figure 4: Vehicle Scrappage Rate by Age



by determining what quantity of these vehicles must be on the road to make up the difference between the known number of cars and light-duty trucks in 1994 and those accounted for by the 1984-1994 vintages. These estimates are derived using the assumption that all vehicle classes scrap at the same rate.

Scrappage rates are based on a logistic function estimated by Miaou (1991 and 1995). The function form and figure plotting the estimated equation are presented below.

$$y_a = \frac{1}{\alpha + e^{-\beta + \gamma a}} + \varepsilon_a \tag{21}$$

Where: y_a = scrappage rate for vehicle of age a
α= 4.60
β = -5.84
γ= 0.44

3.2.1 Declining Vehicle Use With Age

Annual millage by age and vehicle type used by TAFV originated from EPA's Mobile 5 emissions model. Light duty gasoline truck types listed were based on gross vehicle weights <6001 lbs and 6001-8500. Since most light trucks correspond to the lower range, we choose the <6001 type as being representative of both cargo and passenger trucks. Only one light duty gasoline automobile type was used by mobile 5. We choose this type to represent both small and large automobiles. Table below presents the annual mileage by age and type.

Table 7: Annual Miles Driven by Age and Vehicle Type

Age	Small Autos	Large Autos	Cargo Trucks	Passenger
1	14390	14390	15442	15442
2	13612	13612	14508	14508
3	12875	12875	13631	13631
4	12180	12180	12807	12807
5	11522	11522	12032	12032
6	10899	10899	11305	11305
7	10310	10310	10621	10621
8	9751	9751	9979	9979
9	9225	9225	9376	9376
10	8726	8726	8809	8809
11	8254	8254	8276	8276
12	7807	7807	7776	7776
13	7386	7386	7306	7306
14	6987	6987	6864	6864
15	6608	6608	6449	6449
16	6251	6251	6059	6059
17	5913	5913	5693	5693
18	5594	5594	5348	5348
19	5291	5291	5025	5025
20	5005	5005	4721	4721

3.2.2 Starting Initial Stocks of On-Road Vehicles

In order to get the correct number of vehicles purchase in the initial forecast years, the model must be initialized with the correct age distribution of the on-road vehicle stock. The age distribution matters since vehicle scrappage and use rates vary by age. Unfortunately, data do not exist on the age distribution for all 30 years (ages 0-29) of our assumed potential vehicle life. For ages 0-10 corresponding to years 1994 - 1984, we use data from the American Automobile

Manufacturers Association (AAMA, p. 39). We estimate the percentages of the on-road stock for ages 11-29 using the total number of vehicles not accounted for by age 1-10 year old vehicles, the total on-road light-duty vehicle stock, and by making the assumption of a constant historical purchase rate, given the exogenously determined scrappage rate. Defining the multi-year survival rate (i.e., the percentage of vehicles of age surviving) as $\sigma_a = \prod_{\alpha=0}^{a} (1-\gamma_a)$, the number of vehicles of any age *a* on the road at time *t* is the product of the multi year survival rate and the level of investment a years ago.

$$K_{ta} = I_{t-a}\sigma_a \tag{22}$$

If we now sum over all ages of vehicles, this gives us the total number of vehicles on the road at any time t.

$$\sum_{\alpha=0}^{A} K_{ta} = \sum_{\alpha=0}^{A} I_{t-a}\ \sigma_a \tag{23}$$

If we assume a steady-state level of investment, $I_{ta} = I_{t-1,a-1} = I$, then we can factor investment out of the above equation and determine what the steady-state level of investment must have been given our observed level of vehicles and scrappage rate.

$$I = \frac{\sum_{\alpha=0}^{A} K_{t\alpha}}{\sum_{\alpha=0}^{A} \sigma_\alpha} \tag{24}$$

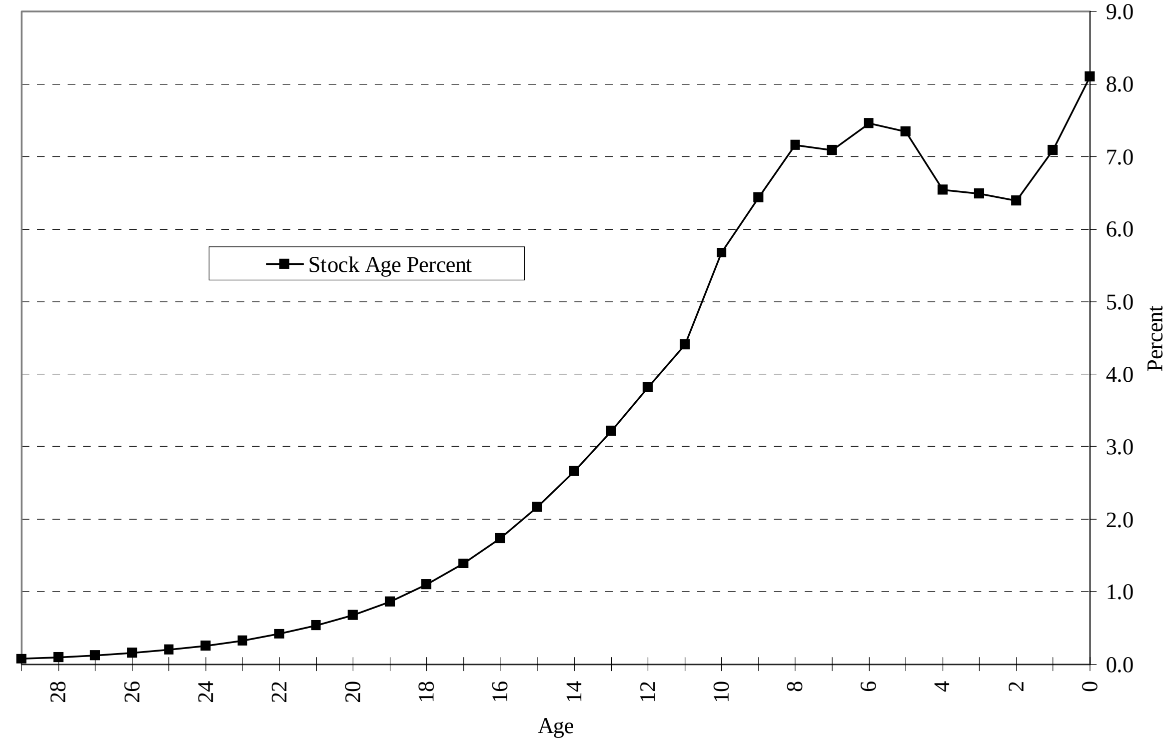
Using this relationship we derive the steady-state level of investment for the 23% of vehicles not accounted for by vehicle of age 0 to 10. Then applying the multi year survival function to the steady-state level of investment we derive a historical age profile for on-road vehicles consistent with our scrappage rate and observed total vehicle stock. Explicitly, we use the following equations.

$$I = \frac{0.23K_{total}}{\sigma_{11}} \quad \text{and} \quad K_a = I \cdot \sigma_a \quad \forall \quad a \geq 11$$

$$where\ \sigma_{11} = \sum_{a=11}^{a=29} = \sigma_a \tag{25}$$

From these calculations the initial percentage on-road vehicles for ages 11 - 29 are calculated. Combining these results with the observed age distribution for vehicles age 0 - 10 yield the age profile shown in Figure 5.

Figure 5: Initial Vehicle Distribution by Age



4.0 FLEET VEHICLES

4.1 Fleet Vehicle Demand

Fleet demand for new conventional and alternative vehicles is derived from exogenous demand for vehicle services by fleet vehicle owners. The mix of new vehicles is also constrained exogenously to reflect the EPACT fleet mandate policy under consideration. Absent EPACT, fleet vehicle owners would freely choose a mixture of conventional and alternative fuel vehicles to meet their demand for vehicle services. Given EPACT, fleet owners are compelled to choose a specified percentage of alternative fuel vehicles in each year. The mandated new AFV purchase percentages vary depending on the policy adopted, i.e., depending on the type of fleets which are subject to the mandate (e.g. federal, state and local government and alternative fuel provider.) In most cases the *mix* of AFVs within the mandated total AFV percentage is unspecified. The percentages of alternative fuel vehicles required to be purchased are given in Table 8 below.

Table 8: EPACT Fleet AFV Purchase Requirements (Percent of Total Vehicle Purchases by Category)					
Year	Alternative Fuel Providers	Federal	State	Early Private and Local	Late Private and Local (if required)
1993	0%	15%*	0%	0%	0%
1994	0%	23%*	0%	0%	0
1995	0%	30%*	0%	0%	0
1996	30%	25%	10%	0%	0
1997	50%	33%	15%	0%	0
1998	70%	50%	25%	0%	0
1999	90%	75%	50%	20%	0
2000	90%	75%	75%	20%	0
2001	90%	75%	75%	20%	0
2002	90%	75%	75%	30%	20%
2003	90%	75%	75%	40%	40%
2004	90%	75%	75%	50%	60%
2005	90%	75%	75%	60%	70%
2006	90%	75%	75%	70%	70%
2007	90%	75%	75%	70%	70%
2008	90%	75%	75%	70%	70%
2009	90%	75%	75%	70%	70%
2010	90%	75%	75%	70%	70%

*These percentages are calculated based upon the EPACT numerical requirements of 5,000, 7,500, and 10,000 vehicle purchased in 1993, 1994, and 1995 assuming 50,000 federal fleet vehicle acquisition per year (Davis and McFarlin, p. 5-5). In addition, federal requirements for 1993 - 1995 have been increased by 50 percent pursuant to Executive Order 12844.

The precise number of alternative fuel vehicles implied by these EPACT requirements has been estimated by EA (1997) based upon the definition of “covered” fleets contained in EPACT. Under EPACT the Secretary of Energy must determine if a late rulemaking is required to attain EPACT’s goals. The late rulemaking would require local governments and private fleets to meet the minimum purchase requirements listed in the table above. Absent the late rulemaking, EPACT’s fleet requirements only include those on federal and state governments and alternative fuel providers. Since the number of vehicles in private fleets are far larger than the all the other fleets combined, the late rulemaking would have a large effect on the required number of AFVs. The estimated number of EPACT fleet vehicles are given in table 9.

Table 9: Estimated EPACT Fleet Vehicle Purchases: No Private Fleet Rule												
	Federal Fleets			**Fuel Providers Non-Electric**			**Fuel Providers Electric**			**Fuel Provider Total**		
Year	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total
1993	3,601	1,816	5,417	17	861	0	0	0	0	17	861	0
1994	2,731	6,056	8,787	17	861	0	0	0	0	17	861	0
1995	2,127	5,266	7,393	17	861	0	0	0	0	17	861	0
1996	2,537	4,210	6,747	427	2,017	0	0	0	0	427	2,017	0
1997	3,018	5,959	8,977	1,662	2,301	3,963	0	0	0	1,662	2,301	3,963
1998	6,388	7,324	13,712	2,871	2,610	5,481	81	775	856	2,952	3,385	6,337
1999	10,770	9,963	20,733	4,214	2,784	6,998	1,116	1,724	2,840	5,330	4,508	9,838
2000	11,296	9,615	20,911	4,214	2,784	6,998	2,036	1,939	3,975	6,250	4,723	10,973
2001	11,296	9,800	21,096	4,284	2,825	7,109	3,063	2,047	5,110	7,347	4,872	12,219
2002	11,296	9,996	21,292	4,354	2,867	7,221	3,063	2,047	5,110	7,417	4,914	12,331
2003	11,296	10,202	21,498	4,424	2,908	7,332	3,063	2,047	5,110	7,487	4,955	12,442
2004	11,296	10,420	21,716	4,424	2,908	7,332	3,063	2,047	5,110	7,487	4,955	12,442
2005	11,296	10,650	21,946	4,557	2,775	7,332	3,063	2,047	5,110	7,620	4,822	12,442
2006	11,296	10,893	22,189	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2007	11,296	11,148	22,444	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2008	11,296	11,419	22,715	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2009	11,296	11,703	22,999	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2010	11,296	12,004	23,300	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441

Table 9 (Continued) Estimated EPACT Fleet Vehicle Purchases: No Private Fleet Rule												
	State Fleets			**Local Fleets**			**Private Fleets**			**Total Fleets**		
Year	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total
1993	333	2,314	2,647	0	0	0	0	0	0	3,951	4,991	**8,942**
1994	333	2,314	2,647	0	0	0	0	0	0	3,081	9,231	12,312
1995	389	2,067	2,456	0	0	0	0	0	0	2,533	8,194	10,727
1996	389	2,067	2,456	0	0	0	0	0	0	3,353	8,294	11,647
1997	984	1,950	2,934	0	0	0	0	0	0	5,664	10,210	15,874
1998	1,000	2,050	3,050	0	0	0	0	0	0	10,340	12,759	23,099
1999	1,314	2,200	3,514	0	0	0	0	0	0	17,414	16,671	34,085
2000	3,826	3,884	7,710	0	0	0	0	0	0	21,372	18,222	39,594
2001	12,055	11,036	23,091	0	0	0	0	0	0	30,698	25,708	56,406
2002	12,220	11,152	23,372	0	0	0	0	0	0	30,933	26,062	56,995
2003	12,387	11,268	23,655	0	0	0	0	0	0	31,170	26,425	57,595
2004	12,557	11,386	23,943	0	0	0	0	0	0	31,340	26,761	58,101
2005	12,729	11,504	24,233	0	0	0	0	0	0	31,645	26,976	58,621
2006	12,903	11,625	24,528	0	0	0	0	0	0	31,926	27,232	59,158
2007	13,079	11,745	24,824	0	0	0	0	0	0	32,102	27,607	59,709
2008	13,257	11,867	25,124	0	0	0	0	0	0	32,280	28,000	60,280
2009	13,439	11,992	25,431	0	0	0	0	0	0	32,462	28,409	60,871
2010	13,623	12,117	25,740	0	0	0	0	0	0	32,646	28,835	61,481

Table 10: Estimated EPACT Fleet Vehicle Purchases: Late Private Rule												
	Federal Fleets			**Fuel Providers Non-Electric**			**Fuel Providers Electric**			**Fuel Provider Total**		
Year	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total
1993	3,601	1,816	5,417	17	861	878			0	17	861	878
1994	2,731	6,056	8,787	17	861	878	0	0	0	17	861	878
1995	2,127	5,266	7,393	17	861	878	0	0	0	17	861	878
1996	2,537	4,210	6,747	427	2,017	2,444	0	0	0	427	2,017	2,444
1997	3,018	5,959	8,977	1,662	2,301	3,963	0	0	0	1,662	2,301	3,963
1998	6,388	7,324	13,712	2,871	2,610	5,481	0	0	0	2,871	2,610	5,481
1999	10,770	9,963	20,733	4,214	2,784	6,998	81	775	856	4,295	3,559	7,854
2000	11,296	9,615	20,911	4,214	2,784	6,998	1,116	1,724	2,840	5,330	4,508	9,838
2001	11,296	9,800	21,096	4,284	2,825	7,109	2,036	1,939	3,975	6,320	4,764	11,084
2002	11,296	9,996	21,292	4,354	2,867	7,221	3,063	2,047	5,110	7,417	4,914	12,331
2003	11,296	10,202	21,498	4,424	2,908	7,332	3,063	2,047	5,110	7,487	4,955	12,442
2004	11,296	10,420	21,716	4,424	2,908	7,332	3,063	2,047	5,110	7,487	4,955	12,442
2005	11,296	10,650	21,946	4,557	2,775	7,332	3,063	2,047	5,110	7,620	4,822	12,442
2006	11,296	10,893	22,189	4,557	2,775	7,332	3,063	2,047	5,110	7,620	4,822	12,442
2007	11,296	11,148	22,444	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2008	11,296	11,419	22,715	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2009	11,296	11,703	22,999	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441
2010	11,296	12,004	23,300	4,557	2,775	7,332	3,170	1,939	5,109	7,727	4,714	12,441

Table 10 (Continued) Estimated EPACT Fleet Vehicle Purchases: Late Private Rule												
	State Fleets			**Local Fleets**			**Private Fleets**			**Total Fleets**		
Year	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total	Cars	LTD	Total
1993	333	2,314	2,647	2,970	5,940	8910	0	0	0	6,921	10,931	**17,852**
1994	333	2,314	2,647	3,029	6,058	9087	0	0	0	6,110	15,289	**21,399**
1995	389	2,067	2,456	3,089	6,179	9268	0	0	0	5,622	14,373	**19,995**
1996	389	2,067	2,456	3,151	6,303	9454	0	0	0	6,504	14,597	**21,101**
1997	984	1,950	2,934	3,214	6,429	9643	0	0	0	8,878	16,639	**25,517**
1998	1,000	2,050	3,050	6,557	3,278	9835	0	0	0	16,816	15,262	**32,078**
1999	1,314	2,200	3,514	6,687	3,343	10030	0	0	0	23,066	19,065	**42,131**
2000	3,826	3,884	7,710	6,821	3,410	10231	0	0	0	27,273	21,417	**48,690**
2001	12,055	11,036	23,091	6,957	3,478	10435	0	0	0	36,628	29,078	**65,706**
2002	12,220	11,152	23,372	9,331	4,665	13,996	50,025	26,212	76,237	90,289	56,939	**147,228**
2003	12,387	11,268	23,655	5,444	23,174	28,618	100,050	52,425	152,475	136,664	102,024	**238,688**
2004	12,557	11,386	23,943	8,277	35,168	43,445	150,075	78,637	228,712	189,692	140,566	**330,258**
2005	12,729	11,504	24,233	9,789	41,509	51,298	175,087	91,744	266,831	216,521	160,229	**376,750**
2006	12,903	11,625	24,528	9,923	41,994	51,917	175,087	91,744	266,831	216,829	161,078	**377,907**
2007	13,079	11,745	24,824	10,059	42,485	52,544	175,087	91,744	266,831	217,248	161,836	**379,084**
2008	13,257	11,867	25,124	10,196	42,982	53,178	175,087	91,744	266,831	217,563	162,726	**380,289**
2009	13,439	11,992	25,431	10,335	43,484	53,819	175,087	91,744	266,831	217,884	163,637	**381,521**
2010	13,623	12,117	25,740	10,476	43,992	54,468	175,087	91,744	266,831	218,209	164,571	**382,780**

As mentioned, in each model run fleet vehicle purchases are derived from the exogenously specified fleet demand for vehicle services. In developing the input data, we work backward from the total fleet vehicle purchases which are potentially subject to the AFV mandate to determine the associated fleet vehicle services demand. In order to convert the yearly EPACT fleet sales into the implied total fleet demand for vehicle services in each year, we accumulate yearly sales into effective stocks assuming that fleets have the same vehicle usage and scrappage profiles as privately purchased vehicles. In addition, we include the existing AFV stocks from recent fleet AFV purchases. These existing stocks of AFVs are shown in the table below and are included in the total demand for fleet vehicle services included in the model.

Table 11: Total Federal, State and Local Government Fleet Alternative Fuel Vehicle Stocks*

Year	1992	1993	1994
	Fleet	Fleet	Fleet
LPG	9420	43032	42585.5
CNG	5687	11782	15942
M-85	2610	7418	10283
E-85	139	387	930
Electricity	0	14	182.5
Total	17856	62633	69923

*Based on EIA, December 1996, Tables 4, 5, 6, pp. 17-18 and EIA, June 1994, Tables 3 and 5 pp. 12-14.

4.2 Fleet Vehicle Choice

Notwithstanding the absolute number (or percentage) of AFVs required under the EPACT requirements, the choice among alternative fuel vehicle types is under the discretion of the fleet owners. In the TAFV model fleets can choose dedicated or dual-fueled vehicles that use alcohol (M85, E85), LPG, CNG, and dedicated electric vehicles. This approach treats fleet vehicle demand as separate category of overall vehicle demand. Fleet vehicles are chosen according the same nested MNL choice model as described in the section on vehicle choice. When fleets choose to purchase dual-fuel vehicles, fleets owners have the second choice over fuel use in each period. Factors influencing the choice of vehicles and fuels include the prices of the vehicles and fuels as well as non-priced attributes. For the present, the fleet valuation of non-priced vehicle attributes is assumed to be the same as those used for non-fleet purchases. Subsequently, fleet vehicle choice parameters may be adjusted to test the sensitivity of the model to these base assumptions.

Two aspects of fleet vehicle use which affect the valuation of non-priced vehicle attributes are whether or not fleet vehicles are assumed to refuel centrally or commercially and the intensity of vehicle use. The model assumes that EPACT fleet vehicles refuel commercially. This has the consequence of providing additional retail fuel availability for privately owned AFVs. The degree to which fleets use commercial refueling facilities can be varied. The second important use difference between fleet and private vehicle use is that fleet vehicles are used more intensely. That is, some fleet vehicles may be driven significantly more per year than private vehicles. This additional driving per year mean that fleets may be more inclined to purchase an AFV in order to take advantage of a small price advantage in an alternative fuel. This factor is not yet reflected in the model.

5.0 VEHICLE PRODUCTION MODEL

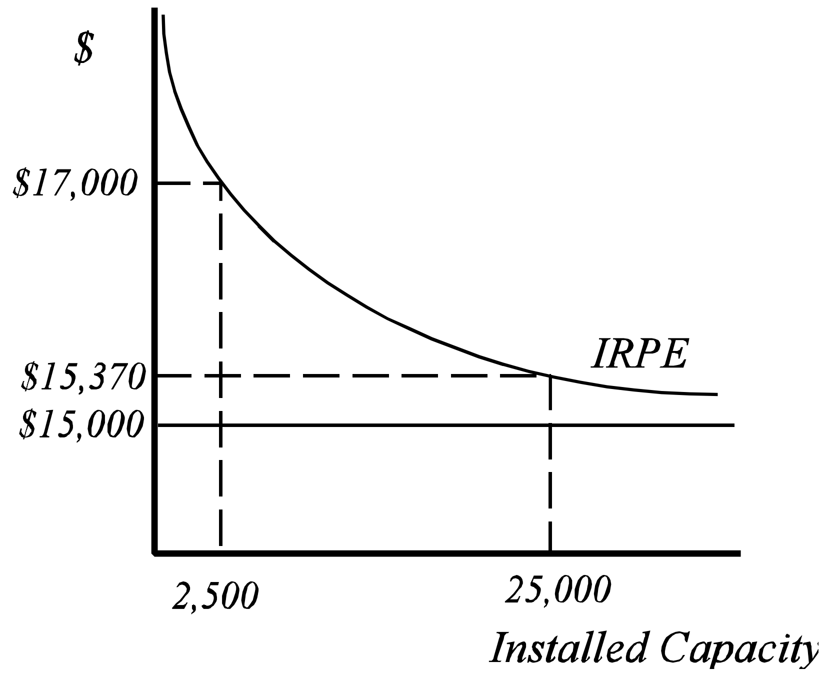
The alternative vehicle supply module currently estimates the retail prices for vehicles in a single size class, as a function of vehicle production, production capacity, and model diversity. The vehicle fuel-types included use the following fuels: gasoline, LPG, CNG, alcohols, and electricity[9]. Both dedicated and dual/flexible fuel vehicles may be produced, including electric hybrid vehicles. This means that, including conventional gasoline vehicles, there are a maximum of 9 vehicle types whose production costs will estimated.

Following EEA (1994, 1995) the costs of AFVs are calculated by using incremental retail price estimates. EEA believes that AFV technologies, except for electric vehicles, are essentially mature. Here "mature" means that further cumulative production will not lead to technological learning and per-unit production cost reductions at a rate significantly faster than conventional vehicle production costs will decline. There do exist, however, per-unit cost savings with large scale production.

Per-unit production costs are modeled as a declining function of the production capacity available in a given year. The volume of production is constrained by the level of cumulative investment (less decay) by manufacturing firms in technology-specific capital, e.g., investment vehicle class and fuel type. Since retail prices are characterized as a function of production volume, the price of vehicles is an endogenous variable. This has the advantages of showing the positive feedback effects from policies (such as AFV fleet programs) that encourage the adoption (and production) of AFVs. The general shape of the incremental retail cost curves are shown below.

[9]Future versions of the model will estimate the retail prices for two car classes and two light truck classes.

Figure 6: General Shape of Incremental Retail Capital Costs



An empirical issue is whether variable costs, in each time period, should be assumed to be constant, or vary with capacity. Data provided by EEA do indicate that average variable costs decline with the level of production. To account for this we subtract out of the variable costs an amount equal to the minimum variable costs at large-scale production (for each fuel and vehicle type.) The remaining portion of variable costs are added to capital costs. We have allocated a portion of variable cost to capital costs such that a portion of variable costs decline with the level of installed capacity. The remaining portion of variable costs is constant.

One key feature of the TAFV model design is to recognize that vehicle manufacturers may underutilize capacity. Thus, the consumer price of vehicles will not necessarily reflect the full cost of capital. Vehicle manufacturers, will nonetheless, still have to pay for all installed capacity. If demand levels in any time period exceed the rated capacity levels of the installed capacity, then the price of vehicles to consumers will also reflect a charge representing a short-run capacity constraints such that incremental vehicle markups will rise.[10],[11]

---

[10]In the AFTM, variable costs for fuels increased at higher demand levels due to feedstock scarcity, but capital charges were assumed to be constant. The same will be true in this model for fuels.

[11]It is possible that low volume production (in the hundreds of units per year) would have unit production costs so high that no vehicles would be demanded. Nonetheless, some vehicles may be sold at a loss by vehicle manufacturers to meet regulatory requirements or as a form of corporate good citizenship. Initially, this behavior will not be explicitly modeled. Scenarios that assume different levels of per-unit vehicle subsidies will be run to examine the importance of this issue.

5.1 Vehicle Diversity Effects on Production Costs

Vehicle production costs functions are defined in the TAFV model for each vehicle fuel-type and size class. As described earlier, consumers gain value from having a choice among a rich set of vehicle models within each size-class/fuel-type. On the other hand, vehicle costs (for each vehicle class and fuel technology) increase as the richness of offerings (number of models) increases. For vehicles, each additional model variant produced will require some amount of specialized capital representing product line fixed costs. Also, for a given level of vehicle production, expanding the number of models will diminish the scale of production for each model, leading to higher costs due to scale diseconomies.

Vehicle diversity is a choice variable under the control of the vehicle producer that reflects the relative richness of models for each vehicle fuel-class type. Diversity is measured by the number of models n_f offered for vehicles of fuel type f. This statistic is adequate to inform the consumer choice module with the simplified make and model representation described by Greene (1995). While model diversity adds to the vehicle producers' costs, there is a motivation for producing diversity since it makes a vehicle type more attractive to consumers. Producers recognize that consumers will be willing to pay for some degree of diversity. The TAFV model solution identifies the market number of vehicle models where the marginal producer cost of increasing diversity balances the marginal consumer benefit of added diversity.

5.2 Mathematical Representation of Vehicle Manufacturers' Behavior

Technological Description

In automobile manufacturing fixed amounts of capital are required before the first unit of production. For the purposes of this analysis, installed capital is modeled as fixed costs at the level of fuel technology and vehicle class. The following equation describes the time-evolution rate of technology-specific capital. Note that vehicle production capital is durable, but not vintaged.

$$K_{(t+1)c} = (1 - \gamma_c)K_{tc} + I_{tc} \qquad \forall\ c, t<T \tag{26}$$

Where

- c conversion process producing vehicles of a particular fuel technology and size class
- γ_c decay rate of installed vehicle production capital type c
- K_{tc} fixed vehicle capital specific to the production of vehicle fuel-type c
- I_{tc} new capital investment.

Vehicle manufacturers are assumed to maximize profits from producing and selling vehicles. Formally, the competitive vehicle manufacturers's economic problem is stated as follows (omitting regional subscripts).

$$
\begin{aligned}
&\underset{Q,N,I}{Max} \sum_t \delta_t \left\{ P_t Q_t - C(Q_t,K_t,N_t) - C^I(I_t) \right\} \\
&\qquad s.t. \quad Q_t \le f(K_t,N_t) \quad \forall t \\
&\qquad K_{t+1} = (1-\gamma_c)K(t) + I_t \quad \forall t<T \\
&\qquad Q_t \ge 0,\ I_t \ge 0 \quad \forall t
\end{aligned}
\tag{27}
$$

where the variables are:

Q_t industry production of vehicles of fuel-type/class c

N_t number of vehicle models offered of fuel-type/class c.

The Lagrangian for this problem is given below.

$$
\begin{aligned}
L = &\sum_t^T \delta_t \left\{ P_t Q_t - C(Q_t,K_t,N_t) - C^I(I_t) \right\} \\
&+ \sum_t^T \lambda^f (Q_t - f(K_t,N_t)) \\
&+ \sum_t^{T-1} \lambda_t^k (K_{t+1} - (1-\gamma_c)K(t) - I_t) \\
&\sum_t^T \mu_t^Q Q_t + \mu_t^I I_t
\end{aligned}
\tag{28}
$$

As described above, vehicle manufacturers face a marginal (and average) cost curve which is downward sloping with respect to installed capital. It is recognized that production of each vehicle type involves some sunk costs and some capital which may be converted to produce other vehicle types. The vehicle price includes a base cost due to "variable" factors. The variable cost includes both the usual variable factors and generic capital costs that are not specific to the production of that vehicle fuel-type and size-class. Generic capital investments are assumed to be recoverable and convertible to the production of other vehicle types. They are treated as a variable cost so that if the production of an AFV is reduced or ended that portion of capital investment is not lost. The vehicle price also includes a capital cost term that is specific to vehicle type and class, c, and that declines with the scale k_c of the vehicle production plant. This fixed cost of vehicle-specific capital is borne even if it is not fully utilized. In addition, vehicle manufacturer's face a charge for offering a diverse number of models, n_c.

Vehicle production costs are represented in the TAFV model, therefore, by three cost components: variable costs, capital costs, and vehicle diversity costs. At the plant-level, the marginal variable costs are constant over output q. That is, per-unit variable costs are given by the constant P_c^{var}:

$$
\frac{C_c^{V-var}(q_c,k_c)}{q_c} = \frac{\partial C_c^{V-var}(q_c,k_c)}{\partial q_c} \equiv P_c^{var}.
\tag{29}
$$

The vehicle incremental costs data that we obtained from EEA (1995(c)) are based on average-cost pricing, and vary with plant scale. We treat them as marginal cost data and construct a cost function such that marginal decisions with respect to vehicle production yield the above marginal pricing outcomes.

For simplicity, the marginal capital charges for vehicle production are fitted to a hyperbolic function which declines with plant scale:

$$P_c^K(k_c^r) = P_c^{kmin} + \frac{B_c}{k_c^r} \tag{30}$$

Here k_c^r is the *plant*-level *rated* production capacity for vehicle type *c*. We use a simple production function where maximum output is maximum capacity divided by the stock-per-flow coefficient (i.e., $q_c \leq f(K) = K/\kappa$). Division of capacity by the stock-per-flow coefficient κ converts a stock of capital into the maximum annual flow of vehicles produced. Since our cost data are specified in terms of rated plant capacity and the model refers to maximum production, we divide by the parameter $\boldsymbol{\rho}$ = 1.15 to convert from maximum capacity to rated capacity.

The cost function is thus derived by integrating the unit variable costs with respect to output levels, and incremental capital costs with respect to installed vehicle production capacity levels. Performing the integration yields the following cost equation, where for each vehicle production plant q_c is the level of output and C^0_c is the constant of integration.

$$c_c^V(q_c,k_c) = q_c P_c^{var} + \left[P_c^{\min} \frac{k_c}{\rho\kappa} + B_c \ln\left(\frac{k_c}{\rho\kappa} \right) + C_c^0 \right] \tag{31}$$

The first term on the right is the plant variable cost, which equal output times the variable costs per unit production. The second reflects the minimum incremental capital cost per unit capacity, achieved for mature-scale levels of plant capital investment K_c. The third term indicates that per-unit incremental capital costs (and hence vehicle prices) decline as the level of installed fixed capital increases from the minimum feasible scale toward mature or full-scale levels of fixed vehicle production capital. Recall that for simplicity the marginal decline in unit costs was a hyperbolic function fitted to our data.

Our approach is to treat each plant (vehicle model) as a separate operation, with capacity roughly equal to the industry average scale for that vehicle fuel-type. Total industry incremental capital costs for the production of each vehicle type are equal to the number of plants times the single - plant capital cost for each plant at average plant scale:

$$C_c^{V-cap}(N_c,K_c) = N_c \underline{C}_c^{V-cap}(K_c/N_c) \tag{32}$$

where

K_c industry-wide production capacity for vehicle type c
N_c number of plant/models for vehicle type c
K_c/N_c average plant capacity.

Total incremental capital costs for vehicle production capacity are calculated here. Since marginal capital costs depend upon the scale of each vehicle production *plant*, the incremental capital cost calculation is a function of total industry capacity (K_{trc}) and the number of vehicle models/plants,

N_{trc}. The minimum incremental marginal capital costs, P_c^{min}, occurs for very large-scale production, and marginal capital costs are greater for lower scale (lower K/N) according to the parameter B_c^V.

$$\underline{C}_c^{V-cap}(K_c/N_c) = \left[\left(P_c^{\min}\frac{K_c}{\rho\kappa_c N_c} + B_c^V \ln\left(\frac{K_c}{\rho\kappa_c N_c}\right)\right) + \underline{C}_c^0\right]\frac{1}{1000} \quad \forall\ c \in C_{VPROD}$$

where

$$\underline{C}_c^0 = -\left(P_c^{\min}\frac{\underline{K}_c^{\min}}{\rho\kappa_c} + B_c^K \ln\left(\frac{\underline{K}_c^{\min}}{\rho\kappa_c}\right)\right) + P_c^{NUMMOD} \tag{33}$$

These are the incremental costs of plant capital specific to vehicle type *c*. Variable and non-incremental (generic) capital costs are captured in the conversion cost terms. Note that total vehicle production stock *K* is divided by ρ=1.15 to convert from maximum production capacity to rated capacity, since the cost functions are specified in terms of rated capacity. $\underline{K}_c^{min}$ is the minimum *plant* capacity for vehicle type *c* (which also equals the minimum industry capacity since the minimum number of plants is one).

Industry-level vehicle production costs for vehicle type and class *c* is equal to the number of models N_c times the plant-level vehicle production costs. The industry-level cost function for vehicle type and class c is given below. The industry collectively and simultaneously determines the output level $q_c = N_c\, \underline{q}_c$, the number of models, N_c and the installed capacity $K_c = N_c\, k_c$

$$\underline{C}_c^V(k_c,q_c) = \underline{C}_C^{V-cap}(k_c) + \underline{C}_C^{V-var}(q_c)$$

$$C_c^V(K_c,Q_c,N_c) = N_c \cdot \underline{C}_c^V(K_c/N_c, Q_c/N_c)$$

$$= N_c\left[P_g^{\min}\frac{K_c}{N_c\rho\kappa} + B_c \ln\left(\frac{K_c}{N_c\rho\kappa}\right) + C_c^0\right] + Q_c P_c^{var} \tag{34}$$

$$= \left[P_c^{\min}\frac{K_c}{\rho\kappa} + N_c B_c \ln\left(\frac{K_c}{N_c\rho\kappa}\right) + N_c C_c^0\right] + Q_c P_c^{var}$$

where C_c^0 is a constant of integration. The variable cost P_c^{var} represents the constant, per-unit variable costs.

Differentiation of the above equation for industry costs with respect to industry levels Q_c and K_c yields the desired marginal conditions, that is, the same marginal costs as those for each individual plant evaluate at the plant levels q_c and k_c:

$$\frac{\partial C_c^V(K_c,Q_c,N_c)}{\partial Q_c} = \frac{\partial c_c^V(k_c,q_c)}{\partial q_c} = P_c^{var}$$
$$\frac{\partial C_c^V(K_c,Q_c,N_c)}{\partial K_c} = \frac{\partial c_c^V(q_c,k_c)}{\partial k_c} = P_c^{kmin} + \frac{B_c}{\rho\kappa(K_c/N_c)} \tag{35}$$

As is shown below, in Section (??) when there in not excess vehicle production capital (at market equilibrium prices) then the producer marginal costs of vehicles are the variable costs plus the marginal capital costs, or:

$$\text{Producer Price} = P_{gv}^{var} + P_{gv}^{kmin} + \frac{B_{gv}}{\rho\kappa k_{gv}}. \tag{36}$$

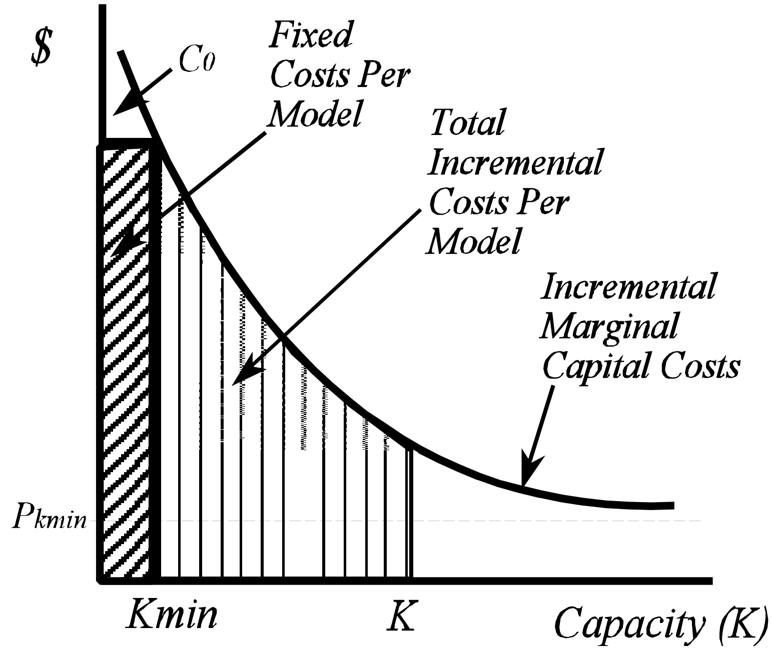
If new vehicle production capital is idle (at market equilibrium prices) then the producer marginal cost of a vehicle is simply the variable cost of vehicles: P_{gv}^{var}.

5.3 Benchmarking Diversity Production Costs

Diversity production costs are properly thought of as costs to the vehicle manufacturer who is offering one or more product lines. These costs are captured by adjusting the scale of production at the plant level to reflect the average size of plants given the number of models produced of a given vehicle type and class. Given N_c models for vehicle type and class c, we calculate vehicle production costs at the industry average scale, K_c/N_c.

We note that the marginal cost function is only defined for plant capacity above the minimum plant scale ($k > k_{min}$), therefore, we interpret the shaded area to the left of K_{min} shown in Figure 7 is the fixed costs per model.

Figure 7: Benchmarking Vehicle Diversity Fixed Costs



Mathematically, we see that this area is defined by splitting the following definite integral for plant capital costs into two parts: one that ranges above and one that ranges below K_{min}.

$$
\begin{aligned}
c_c^{V-cap}(k_c) &= \int_0^{k_c} \left\{ P_c^{kmin} + \frac{B_c}{\rho\kappa x_c} \right\} dx \\
&= \int_{Kmin}^{k_c} \left\{ P_c^{kmin} + \frac{B_c}{\rho\kappa x_c} \right\} dx + \int_0^{Kmin} \left\{ P_c^{kmin} + \frac{B_c}{\rho\kappa x_c} \right\} dx
\end{aligned}
\tag{37}
$$

Performing the integration over plant capacity k_c yields:

$$
N_c c_c^{V-cap}(k_c,q_c) = N_c \left\{ \left\{ \frac{P_c^{k\infty} k_c}{\rho\kappa} + B_c \ln\left(\frac{k_c}{\rho\kappa}\right) \right\} - \left\{ \frac{P_c^{k\infty} k_c^{kmin}}{\rho\kappa} + B_c \ln\left(\frac{k_c^{kmin}}{\rho\kappa}\right) \right\} + c_c^0(k_{\min}) \right\}
\tag{38}
$$

The first term on the right-hand-side represents the total incremental capital costs per model that vary with plant size and are a function of the minimum plant size. The latter two terms represent fixed (sunk) costs per model that are independent of plant capacity, but reflect the presumed minimum size of each plant. We assume that fixed costs per model are equal to the fixed capital expense of a plant with a minimum scale of k_c^{min} (1000) vehicles.

Substituting in average plant capacity K_c/N_c for plant scale k_c, we get the following expression.

$$C(K_c,Q_c,N_c) \;=\; N_c\left\{\left\{\frac{P_c^{kmin}K_c}{N_c\rho\kappa} + B_c\ln(\frac{K_c}{N_c\rho\kappa})\right\} - \left\{\frac{P_c^{kmin}k_c^{\min}}{\rho\kappa} + B_c\ln(\frac{k_c^{\min}}{\rho\kappa})\right\} + c_c^0(k_c^{\min})\right\} \tag{39}$$

Multiplying through by N_c and simplifying we get:

$$C(Q_c,K_c,N_c) \;=\; P_c^{var}Q_c + \left\{\frac{P_c^{k\infty}K_c}{\rho\kappa} + N_cB_c\ln(\frac{K_c}{N_c\rho\kappa})\right\} - N_c\left\{\frac{P_c^{k\infty}k_c^{kmin}}{\rho\kappa} + B_c\ln(\frac{k_c^{kmin}}{\rho\kappa})\right\} + N_cc_c^0(k_{\min}) \tag{40}$$

We are interested in the marginal cost of supplying another model, that is of increasing the number of plants N_c. Taking the derivative of the above equation for total industry costs with respect to the number of plants/models N_c we get:[12]

$$\frac{\partial C_c^V(K_c,Q_c,N_c)}{\partial N_c} \;=\; B_c\left(\ln\left(\frac{K_c}{N_c\rho\kappa}\right) - 1\right) - \left\{\frac{P_c^{k\infty}k_c^{kmin}}{\rho\kappa} + B_c\ln(\frac{k_c^{kmin}}{\rho\kappa})\right\} + c_c^0 \tag{41}$$

Since there is not guarantee that the first term is positive, as plant scale K_c/N_c varies, this marginal condition can be negative when the integration constant c_c^0 is omitted. Since industry costs rise with the number of models produced, and from the geometry of the problem shown below, we can interpret c_c^0 as the industry level fixed cost of per model. The marginal cost of producing diversity depend on the level of industry scale K_c, since it clearly costs more to increase the number of vehicle models when the average model production is larger. Therefore, these fixed costs are benchmarked such that at the minimum plant size, $k_c^{\min}$, the marginal costs of adding a new plant is zero, and increasing in magnitude for larger plant sizes. If we evaluate the marginal costs of increasing the number of models at the point $\frac{K_c}{N_c} = k_c^{\min}$, then we get the following expression.

$$\left.\frac{\partial C_{gv}^V(Q_{gv},n_{gv},K_{gv})}{\partial n_{gv}}\right|_{\frac{K}{n}=k^{\min}} \;=\; -B_{gv} - \frac{P_{gv}^{k\infty}k_{gv}^{kmin}}{\rho\kappa} + c_{gv}^0 \;\equiv\; 0. \tag{42}$$

[12]Note that in taking this derivative we are holding K (= n k) fixed with respect to n since we are trying to determine industry-level changes in costs (for a given vehicle type and size class).

The first two terms on the immediate right-hand-side of the equation, $B_{gv} + \frac{P_{gv}^{k\infty} k_{gv}^{kmin}}{\rho\kappa}$, give us the formula for calculating the marginal cost of capital at the minimum plant size times the minimum plant capacity. This is area shown in the rectangle shown in ?, ?. Thus, c_c^0 is chosen such that the three terms in 43 exactly offset one another.

Note that total capital costs and marginal costs with respect to N_c are independent of the choice of units for capital, k_c. Any change in units adjusts the upper and lower limit terms of the integral of marginal capital costs in a way which cancel out. This is as it should be. Note also that the proper units for the terms are:

$P^{k\infty}$	annual capital charge per unit capacity ((dollars per (vehicle/year))/year)
k	capacity per plant ((vehicles/year)/plant)
K	capacity industry-wide (vehicles/year)
c_c^0	annual capital charge per plant (dollars per plant/year)
B	annual capital charge per plant (dollars per plant/year)

5.3.1 Lags and Lead-times for Vehicle Production Capital Investments

In principle we need to know the lag between the decision to invest in new vehicle production capacity and the time at which vehicles can be produced. We would also need to know the magnitude of up-front investment required per unit of capacity, by vehicle type. The actual process by which investments are made over time and alternative fuel vehicle are produced is undoubtedly complicated. Some vehicle types may require a simple variation in production line setup (e.g. alcohol FFVs), some may require a moderate investment in new equipment (CNG vehicles), while others may require a protracted investment in a new facility (electric vehicles). For a simplified model significantly less information is necessary.

The model can accommodate three types of production investments:

"Putty" investments are quick and flexible, essentially allowing variable short-run capacity. For this type of production, the amount of capacity is assumed to vary smoothly with the current level of production, and the per-unit capital costs can be rolled into the variable costs.

"Durable Quick" investments can be made with no lead-time, but last more than one period, and depreciate gradually. In this case the vehicle production "capacity" is never strictly binding upper limit on output. However, investments will be made only if the current and expected future profitability is adequate.

"Durable Lagged" investment are available for production only after a delay, which for simplicity we take to be one period. In this case, current capacity can be a binding constraint on output. Unit variable costs can be fixed or rise sharply as maximum capacity is approached. These investments are made based only on the expectation of future returns.

Vehicle production capacity is treated as the last of these three capital types.

5.4 Data on Vehicle Prices and Characteristics

Manufacturers' suggested prices for 1995 model year vehicles by manufacturer, division and model are derived from Automotive News Market Data Book 1995, pages 63-67. The average model price (Table 7) was derived using the average of the lowest and highest submodel type. These were then weighted by model sales (pp 19-38) to arrive at the industry average car and truck prices.

Table 12: Conventional Vehicle Manufacturers' Suggested Retail Price and Sales Data

Vehicle Type	$94 Vehicle Prices (1995 models)	Vehicle Sales (calendar 1994)
Small Autos	$16,639	4,996,018
Large Autos	$22,504	3,529,832
All Autos	$19,067	8,525,850
Light Duty Cargo Trucks	$16,423	3,285,223
Light Duty Passenger Trucks	$21,424	2,782,984
All Light Duty Trucks	$18,717	6,068,207
All Autos and Light Duty Trucks	$18,921	14,594,057

Source: Automotive News, 1995, pp. 63-67.

5.5 Used Vintaged Vehicle Valuation at Terminal Time

5.5.1 Motivation

Since the period of analysis in TAFV is finite (years 1996-2010), care must be exercised to properly treat the final period. In a dynamic model certain decisions have long-lasting effects. In particular, investments in durable capital influence capital stocks, prices, and profits for many subsequent years. The dynamic model seeks to characterize the competitive decisions by firms and consumers in making capital investments, which will naturally depend on the expected revenue to be gained from that capital in subsequent years. As the model analysis loops over years and approaches the final year in the finite horizon, some fraction of newly purchased durable capital will be expected to survive past the terminal time. The problem is to specify the "terminal value" or "final value" of surviving capital in the terminal period, so that the resulting private investment decisions are reasonable and follow a smooth path in the immediately preceding years.

For vintaged capital stock such as vehicles, the terminal value problem is somewhat complicated by the fact that each vehicle age will have a different final value, depending on its remaining life expectancy and usefulness. The following sections briefly describe the theory and method used to establish the final value of vehicle stocks. A more exhaustive treatment is included in Appendix 2.

5.5.2 Implied Valuation of Vintaged Capital Stock in Discrete-time Finite-Horizon Dynamic Model

Our objective is to exogenously specify the salvage value of vintaged capital stock (used vehicles) in the terminal time period. We wish to be consistent with the underlying vehicle purchase behavior in TAFV. To achieve this, it is first useful to see how capital stock is valued by a discrete-time finite time-horizon dynamic model. After stating the vintaged stock equations of TAFV model formally, and computing the optimality conditions, (i.e., market equilibrium conditions, see Appendix 2), we arrive the unsurprising result below. In each period, the shadow value of capital stock V_{Kta}^{tot} is just equal to the NPV of all remaining marginal use-value B'_{Ut} plus the discounted salvage value $F_{t,a+T-t}$ of any portion of the stock surviving until the terminal period.

$$V_{Kta}^{tot} \equiv \begin{cases} \displaystyle\sum_{\alpha=a}^{a+T-t} \delta_{t+\alpha-a} \left(\frac{B'_{U,t+\alpha-a} - C'_{U,t+\alpha-a}}{\kappa_\alpha} \right) \frac{\sigma_\alpha}{\sigma_a} + \delta_T F_{T(a+T-t)} \frac{\sigma_{a+T-t}}{\sigma_a} & T-(A-a) \le t \le T \\ \displaystyle\sum_{\alpha=a}^{A} \delta_{t+\alpha-a} \left(\frac{B'_{U,t+\alpha-a} - C'_{U,t+\alpha-a}}{\kappa_\alpha} \right) \frac{\sigma_\alpha}{\sigma_a} + 0 & t \le T-(A-a) \end{cases} \tag{43}$$

For vehicles, the marginal use value is comprised of the marginal benefit of vehicle services less the marginal effective fuel costs. This used vehicle valuation is dependent on vehicle lifetime A, discount rate δ, survival rates σ_a, and marginal productivity $1/\kappa_a$ (use rate for vehicles). In sum, a dynamic, finite time horizon model (such as the TAFV) establishes an equilibrium value for vintaged capital that reflects the discounted stream of remaining use values, plus the salvage value of the remaining fraction of the stock in the terminal period, if any. Thus, if we knew the marginal use value for vehicle stock in each year, we could pre-calculate the terminal salvage value for stock of each age.

5.5.3 Equilibrium Annual Net Use Value for Capital

The previous review shows how a dynamic optimization model such as the TAFV model implicitly values capital of age a based on its discounted stream of annual use-values for its remaining lifetime. In seeking to calculate exogenous salvage values for the model, we need annual marginal-use values. Our problem is to avoid circularity: marginal use values are endogenous to the model, and will reflect any exogenously specified salvage values. Thus we need another tack to establish marginal use value, and thereby, salvage values. This section derives an estimate of the stationary *equilibrium* annual use value for vintaged capital in terms of initial capital cost. This annual use value estimate, in turn, is used to construct an estimate of the equilibrium salvage value of vintaged vehicle stock. The estimate of equilibrium salvage value is used in the TAFV model for final valuation of the remaining vehicle stock in the terminal time period. In the fully dynamic model, investments in each period are based on knowledge of capital use-value in the current and all future periods. Our method of final valuation promotes smooth and consistent vintaged-capital investment (vehicle purchase) behavior in the dynamic model, even as the time approaches the terminal period. For the myopic (recursive period-by-period) solution approach, investments in each period are based on current use-value and an expectation of cumulative future value. Our method of vintaged stock valuation is also useful in the myopic solution approach for constructing a myopic estimate of cumulative future vehicle use-value in each period.

Efficient consumer behavior will lead to purchases of new vehicles up to the point where the discounted stream of marginal use benefits for a new vehicle (net of fuel and operating costs) equals the cost of a new vehicle. If the new vehicle cost is roughly constant, and if consumers continue to purchase new vehicles, then this relationship will hold for every time *t*. In a stationary model, we would expect marginal undiscounted use-benefits to approach a constant value. In this case, Appendix 2 shows that equilibrium annual marginal use net benefits will adjust until:

$$B_U^{\prime} - C_U^{\prime} = \frac{C_0}{\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \tag{44}$$

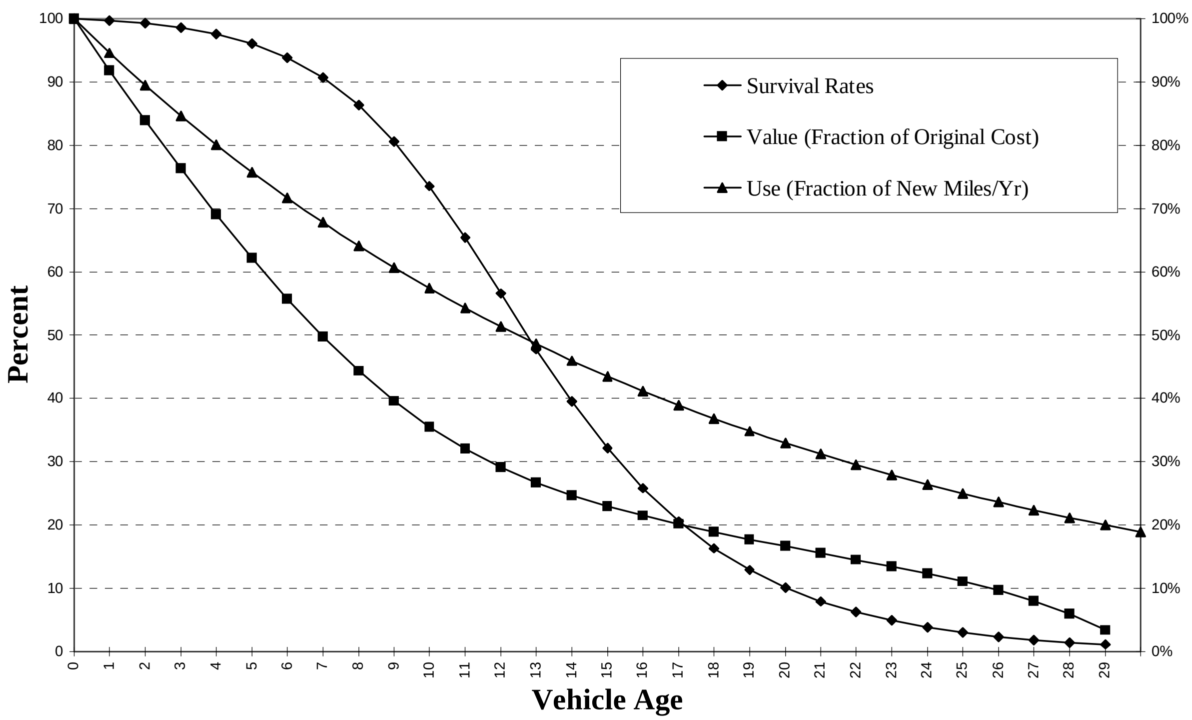
5.5.4 Exogenous Construction of Terminal Period Salvage Values for Used Capital Stock

We use this steady-state estimate of marginal use value to construct terminal stock salvage values. Substituting in the expression for "equilibrium" marginal use-benefit *B'* based on new vehicle cost from equations (44), (135), we have an expression for the equilibrium value of used vintaged capital stock in terms of its current age *a*, initial cost C_0, future survival rates σ_α and future marginal productivity $1/\kappa_\alpha$:

$$V_a = C_0 \frac{\frac{1}{\delta^a \sigma_a} \sum_{\alpha=a}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}}{\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \tag{45}$$

All of these parameters are exogenous except vehicle cost, which varies with production scale. We estimate exogenous salvage values using large-scale production costs. Note that the capital survival rate σ_α and marginal productivity $1/\kappa_\alpha$ (inverse stock-per-flow) appear together and could be combined into a single age-related factor. For vehicles, this means that vehicle scrappage rates and declining use-rates with age are largely interchangeable, at least in terms of vehicle valuation. The resulting salvage value of vehicles, as a fraction of their original cost, is shown in Figure 8.

Figure 8: Vehicle Survival Rates, Declining Use Rates, and Used Vehicle Value



5.6 Data: Vehicle Production Costs Versus Production Scale

Vehicle production cost parameters are drawn from the data and accounting methodology in EEA (1995). EEA believes that most, if not all, alternative fuel vehicles will be "derived" from gasoline vehicles "in that they will use the same engine, drive train and body (except for electric vehicles) as conventional cars." (EEA 1995c: 1-1). Given these expectations, the necessary vehicle components and their costs were identified for each size class and fuel technology. The incremental costs were estimated compared to a conventional gasoline vehicle of nearly identical interior room, features, and performance (within 10 percent). To the extent that vehicle ranges will differ, the TAFV model accounts for range through non-price consumer cost terms in the multinomial vehicle choice framework. In the TAFV model, and in the EEA analysis, vehicle manufacturers are assumed to behave competitively, i.e. to charge a price that earns a normal rate of return.

The EEA vehicle production costs were partitioned into incremental fixed (capital) and variable costs for three different production plant capacities. The capital cost parameters for equation ? were fitted through per-vehicle incremental capital cost for three plant sizes. The Table below shows the resulting incremental vehicle production costs at those sizes, using the TAFV model's fitted parameters.

Table 13: Cost Data for Vehicle Production and Fuel Retailing

Incremental Vehicle Production Costs
(Capital and Variable, Compared to a Gasoline Vehicle)*

	Plant Scale (Vehicles per Year)		
Vehicle Type	2,500	25,000	100,000
Alcohol Dedicated	$2,038	$363	$223
Alcohol Flexible	$1,911	$409	$284
CNG Dedicated	$5,349	$1,841	$1,548
CNG Dual	$5,792	$2,015	$1,701
LPG Dedicated	$3,745	$972	$741
LPG Dual	$3,778	$1,109	$887
Electric Dedicated (1996)	$42,125	$11,060	$8,471
Electric Dedicated (2010)	$29,627	$5,974	$4,003

*For large passenger vehicles. Note: these figures reproduce the estimated IRPE based on EEA's accounting methodology, "Specification of a Vehicle Supply Model for TAFVM," Sept., 1995, p.1-2. They differ slightly from some numbers in EEA's Table 5-2.

6.0 FUEL RETAIL SECTOR DETAIL

6.1 General Characteristics of Retail Supply

The motor fuel retail supply module is designed to capture the cost of retailing the various motor fuels. All fuels, except electricity, are assumed to be sold at commercial retail fuel outlets. Cost estimates supplied by EEA are based on a convenience store selling 150,000 gallons of gasoline per month with monthly operating costs of $27,334 of which $15,000 comes from non-fuel sales (EEA, 1995a, p. 1-1). Alternative fuel retailing capital and markup costs are calculated given current and 2010 technologies assuming 50,000 and 25,000 GGE levels of station conversions (see Table 14).

A key variable to be determined is retail fuel availability, σ^R_{tf}, the fraction of retail stations offering fuel *f* in year t. If retail fuel availability for a particular fuel is low, then consumers will bear additional travel time costs to refuel. Consumers, therefore, can be expected to tradeoff additional travel time costs for refueling with higher per GGE costs for fuel. The retail sector is designed to be able to accommodate this tradeoff by allowing fuel retailers to maintain additional retail availability by increasing capacity in low volume fuels by bearing additional expenses equal to the cost of spreading out the retail fuel infrastructure costs over a lower output volume. Retail fuel availability is thus endogenous to the retail model.

Table 14: Current Capital and Markup Costs for Fuel Retailing*

Fuel Type	Gasoline	M85	E85	LPG	CNG
Capital Cost @ 50,000 GGE	N.A.	$162,040	$162,040	$259,719	$927,100
Capital Costs @ 25,000 GGE	N.A.	$106,620	$106,620	$168,710	$761,814
Markup $/GGE @ 50,000 GGE	0.082	$0.121	$0.121	$0.144	$0.418
Markup $/GGE @ 50,000 GGE	0.082	$0.133	$0.133	$0.162	$0.560

*Source: EEA, 1995b, Table 1. Interest rate for added costs: 7.5%.

Table 15: 2010 Capital and Markup Costs for Fuel Retailing*					
Fuel Type	Gasoline	M85	E85	LPG	CNG
Capital Cost @ 50,000 GGE	N.A.	$148,600	$148,600	$192,800	$876,400
Capital Costs @ 25,000 GGE	N.A.	$99,875	$99,875	$135,300	$734,600
Markup $/GGE @ 50,000 GGE	0.082	$0.118	$0.118	$0.128	$0.406
Markup $/GGE @ 50,000 GGE	0.082	$0.130	$0.130	$0.146	$0.547

*Source: EEA, 1995b, Table 1. Interest rate for added costs: 7.5%.

There are some other important assumptions which characterize the retail sector. In particular, fuel distribution capacity is added in variable quantities with a minimum installation requirement of 16.67 % (one of six pumps) and priced to cover the full costs of capacity increment even though capacity utilization may vary. It is further assumed that there is no lead time for capacity expansion decisions since the time-step of this model is one year and retail capacity can be expanded within that time period. Total retail capacity, once installed, remains in place subject to depreciation. This feature allows firms to build, and pay the consequence for, excess capacity. In addition, the fraction of station capacity dedicated to each fuel is restricted to not increase or decrease by more than 15% each time period. Retail capital costs are amortized into annual fuel sales, and are not accrued at the time of installation. This assumption is consistent with assuming that there are no excess or sub-normal short-run profits. Less than full utilization of capacity is possible provided normal rates of return are achieved by increasing per-unit markups.

6.1.1 Full or Partial Utilization of Retail Station Capacity

As mentioned above, the retail model is specifically designed to allow for less than full utilization of retail capacity. This feature provides a mechanism to gauge the effectiveness of subsidies or other policy levers which could promote availability and reduce the search and travel time of refueling. In order to allow for partial utilization of retail refueling capacity it is necessary to derive a relationship between retail fuel availability, σ^R_{tf}, (fraction of stations offering fuel *f* at time t, e.g., station share), fuel demand shares, σ^D_{tf}, (fraction of total demand for all fuels, provided by fuel *f*) station capacity shares, θ_{tf}, (the fraction of each station's pump capacity dedicated to fuel *f*) the utilization rate of station capacity for fuel *f*, u_{tf}, the overall utilization rate of all stations capacity, u_t, the number of retail stations offering fuel *f*, N_{tf}, and the total number of retail stations, N_t, in any given year t. As a first step, retail fuel ability for fuel *f*, σ^R_{tf}, is defined by the following expression.

$$\sigma_{tf}^{R} = \frac{N_{tf}}{N_t} \tag{46}$$

Making the simplifying assumption that all stations are of equal fuel capacity, $\underline{K}_t$, (on a BGE basis) then the total retail capacity is given as: $K^R_t = N_t\underline{K}^R_t$. Stations offering fuel *f* dedicate fraction θ_{tf} of their capacity to fuel *f*. Thus, we know that the capacity for fuel *f* is given by:

$$K_{tf}^{R} = N_{tf}\underline{K_t}^{R}\theta_{tf} = \frac{N_{tf}}{N_t}K_t^{R}\theta_{ft} = \sigma_{tf}^{R}K^{R}\theta_{tf} \tag{47}$$

Using this construction, one sees that θ_{tf} should be interpreted as the average station capacity share for fuel *f*. It is also true that for fuel *f*, the relationship between retail capacity, K_{tf}, and the utilization of that capacity can be derived by noting the alternative definition of retail fuel availability.[13]

$$\sigma_{tf}^{R} = \frac{u_t}{u_{tf}}\frac{\sigma_{tf}^{D}}{\theta_{tf}} \tag{48}$$

The equality of both expressions for σ_{tf}^{R} can be seen letting q_{tf} be the chosen retail supply of fuel *f* q_t be the total quantity of fuel supplied, and by noting that:

$$\frac{u_t\sigma_{tf}^{D}}{u_{tf}} = \left(\frac{\sum q_{tf}}{\sum K_{tf}^{R}}\right)\frac{K_{tf}^{R}}{q_{tf}}\frac{q_{tf}}{q_t} = \frac{K_{tf}^{R}}{K_t^{R}} \tag{49}$$

These alternate definitions for σ_{tf}^{R} can be used to derive a cost function for producing fuel and fuel availability (convenience). Let $\underline{C}_f(\theta_f)$ be the annualized cost per unit retail capacity ($/BGE) of having θ_{tf} percentage of a station's pumps dedicated to fuel *f* in year t. Accordingly, for total retail supply of fuel *f* equal to q_{tf} ($q_{tf} \le K^R_{tf}$) the total markup is derived below.
These relationships show the total retail cost of supplying quantity q_{tf} of fuel *f* in year t.

$$C^{R}(\sigma_{tf}^{R}, q_{tf}^{R}, K_{tf}, K_{tT}) = \underline{C_{tf}}(\theta_f) * K_{tf} = \underline{C_{tf}}(\theta_f)\left(\frac{u_t}{u_{tf}}\right)q_{tf} = \underline{C_{tf}}^{R}(\theta_f) * K_t * \theta_{tf} * \sigma_{tf}^{R} \tag{50}$$

This form of the cost function shows that total (and per-unit) costs increase as capacity for a fuel exceeds its demand (third term from the left). Retail firms will adjust θ_{tf}, σ^R_{tf}, K^R_{tf}, and u_{tf} to maximize their profit given demand for fuel and availability. Less than full pump capacity utilization can occur provided that consumers demand fuel availability enough to compensate retailers for excess capacity. It is assumed that all stations experience the same average utilization.

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[13] It can be shown that this equation, along with the other constraint equations given below, insures that the following condition is met: $\sum_f \sigma_{tf}^{R}\theta_{tr}^{R} = 1$. This equation says that the sum of the retail availability of each fuel weighted by the average retail capacity devoted to each fuel equals one.

This formula is operationalized by taking into consideration a given reference fraction of pumps dedicated to each fuel, $\overline{\theta}_{tf}$, and a bench marking parameter η^R. The operational formula for *total retailing cost* for fuel *f* is given below.

$$C^R(K_{tf}^R,\theta_{ft}^R) = K_{tf}^R * \underline{C}_{tf}\left(\frac{\overline{\theta}_f^R}{\theta_{ft}^R}\right)^{\eta}, \quad \forall\ t,f \tag{51}$$

When the chosen fraction of pumps dedicated to fuel *f*, θ_{tf}^R, is equal to the reference function, $\overline{\theta}_{tf}^R$, then the operational cost function reflects the costs given in the reference data shown in Table 14. The data in Table 14 give unit markup costs for two different levels of station conversions, 25,000 and 50,000 GGE per month. Hence, the benchmarking parameter η, which can be interpreted at the price elasticity of retail supply, is derived from the data in based upon the data displayed in Table 14 using the relationship: $C_{tf}^{low}\cdot\left(\frac{\theta_{tf}^{high}}{\theta_{tf}^{low}}\right)^{\eta} = C_{tf}^{high}$, where C_{ft}^{high} is the unit cost associated with a conversion fraction, θ_{tf}^{high} , of 50,000 GGE and so forth. Solving this equation we derive the relationship

$$\eta_{tf} = \frac{\ln(C_{tf}^{high})/(C_{tf}^{low})}{\ln(\theta_{tf}^{high})/(\theta_{tf}^{low})}. \tag{53}$$

It is also worth noting the *per-unit* retail costs are given by

$$\frac{C^R}{q_{tf}^R} = \frac{K_{tf}^R * \underline{C}_{tf}(\overline{\theta}_f^R/\theta_f^R)^{\varepsilon}}{q_{tf}^R}, \quad \forall\ t,f. \tag{54}$$

By substitution, we get the following form which shows the explicit dependence of per-unit retail costs on the level of capacity utilization.

$$\frac{C^R}{q_{tf}^R} = \frac{\underline{C}_{tf}(\overline{\theta}_f^R/\theta_f^R)^{\varepsilon}}{u_{tf}^R}, \quad \forall\ t,f \tag{55}$$

6.2 Retail Fuel Demand

Drivers have a demand for transportation services which can be met by a variety of fuel and vehicles. The demand for fuel arises from a demand for the benefits that the fuel provides. Fuel demand, like the demand for any other good, is determined by the price of the good, the price of substitute goods and a host of socio-economic factors. The price of a fuel includes both the purchase price (measured in GGEs) and non-market price attributes. The most important non-market cost associated with fuels is the cost of availability. By availability cost we mean the cost that drivers must incur in terms of travel distance and time to refuel.

In a world with viable alternative fuels and vehicles some vehicles will be still dedicated to a particular fuel type while other flexible, vehicles will be able to use one or both of two fuels. Whether a dedicated or flexible fuel vehicle is purchased depends on the price of the vehicle and the stream of expected fuel prices. With dedicated vehicles the quantity of fuel purchased depends on its own price. With flexible fuel vehicles the choice over two competing fuels depends on the relative market and non-market prices of those two fuels.

To model the cost of fuel availability we follow the approach of Greene (1997) who models availability using a random utility, binomial logit choice framework. Within this framework, the value, or utility, that the jth individual receives from choosing fuel option i is given by

$$U_{ij} = A_i + B \cdot P_i + C \cdot g(\sigma_i^R) + \varepsilon_{ij} \tag{56}$$

where A_i are non-price attributes of the fuel (e.g., safety, smell, etc.), P_i is the price of the fuel, $G(\sigma_i^R)$ is the perceived retail availability of the ith fuel and ε_{ij} is a random error term. The term B converts the market price of fuels in to consumer satisfaction or utility and, hence, can be interpreted as the marginal utility of a dollar. The log of the odds in favor of purchasing fuel option 2 rather than fuel option 1 is given as[14]

$$\ln\left(\frac{Prob_2}{Prob_1}\right) = A_2 - A_1 + B(P_2 - P_1) + C(g(\sigma_2) - g(\sigma_1)). \tag{57}$$

To determine what percentage of the time consumers would choose to use one fuel verses another given different fuel prices and availabilities, Greene asked the following question in two national surveys:

> "Suppose your car could use gasoline or a new fuel that worked just as well as gasoline. If the new fuel costs 25 (10/5) cents LESS per gallon but was sold at just one in 50 (20/5) stations, what percent of the time would you buy this new fuel?"

The results from these surveys were used to estimate (57). In order to do the estimation, a functional form must be chosen for $g(\sigma)$. Greene estimated four forms: linear - $g(\sigma) = \sigma$, exponential - $g(\sigma) = e^{b\sigma}$, power - $g(\sigma) = \sigma^b$, and logarithmic - $\ln(\sigma)$. The costs per gallon for limited fuel availability using these different functional forms are shown in Figure 9 below.

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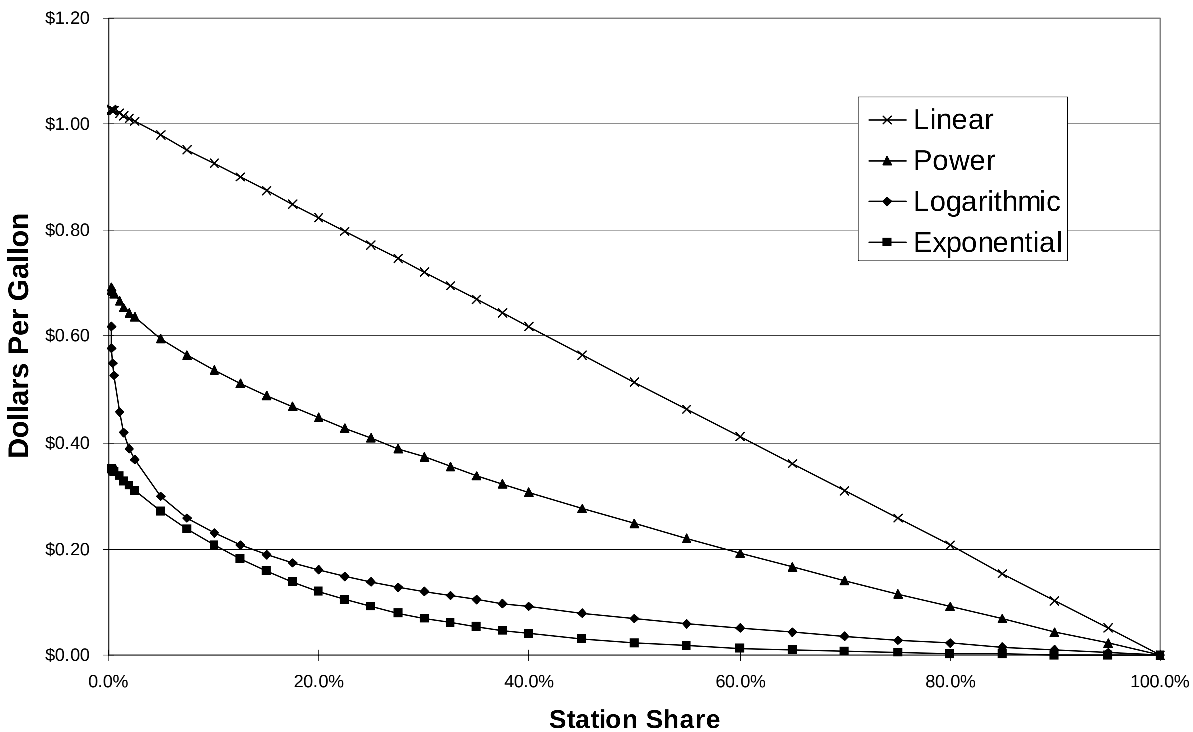
[14]This result follows from making the standard assumption that the error term follows a type 1 extreme value distribution, see Madalla, Chapter 2.

Figure 9: Costs of Limited Retail Fuel Availability



Greene notes (p. 34) that it is not possible to definitively discriminate amount the alternative functional forms, but that the exponential functional form fits the data best and behaves reasonably over the whole range of fuel availabilities. Besides issues of fit, we have chosen to use the exponential functional form because our intuition tells us that at 50% fuel availability (every other gas station) the cost penalty ought to be small. For the exponential functional form, the cost penalty at 50% availability is 2¢ per gallon, the next lowest fuel availability cost is 7¢ per gallon found using the logarithmic functional form. At 0.1% fuel availability the cost per gallon, using the exponential functional form, is 35¢.

In the TAFV model, this cost of fuel availability is operationalized by using the following expression for the total costs of availability for each fuel.

$$C^{A}_{trc}(\sigma^{R}_{trc}, a_{trc}) = \left(C^{Aref}_{trc} e^{b(\sigma_{trc} - \sigma^{ref}_{trc})} - C^{Aref}_{trc} \right) \frac{Q^{R}_{gaso}}{Q^{R}_{C}} a_{trc} \quad \text{for } c \text{ a retail process } (c \in C_{RET}) \qquad (58)$$

These costs are linear in the amount of each fuel retailed (a_{trc}, the level of activities of the retail process). Here C^{Aref} is the cost per gallon at the reference availability σ^{ref}, and b is the elasticity of availability cost with respect to availability share. Subtracting C^{Aref} assures that the incremental availability cost is zero at complete availability (σ=1). The relative frequencies of refilling due to the differing energy densities of fuels and possible differing ranges is reflected by the ratio of gallons per gasoline refill to the number of gallons of gasoline equivalent per refill with fuel *c*, Q^R_{gaso}/Q^R_c.

6.3 Retail Constraint Equations

In order to operationalize the retail sector, a number of other equations and parameters are necessary. For completeness, all of the equations and parameters for the retail supply section are given below.

Fuel Outputs Equal Conversion Activity Levels
The quantity of each motor fuel retailed equals the level of the retail conversion process.

$$q^R_{tf} = a^R_{tf}, \quad \forall \quad t,f \tag{59}$$

Output Must be Less Than or Equal to Capacity
Since retail station capacity is treated as a durable stock, the quantity of each motor fuel retailed must be less than the current retail capacity. This allows retail capacity to exceed retail quantity when cost of excess retail capacity is less than the value of increased fuel availability.

$$q^R_{trc} \le K^R_{trc} \quad \forall t,r,c\varepsilon C_{RET} \tag{60}$$

Total Retail Capacity
The total retail capacity equation tracks the retail capacity for all fuels, so that retail availability for each fuel may be calculated.

$$K^R_{tr} = \sum_{c\varepsilon C_{RET}} K^R_{trc} \tag{61}$$

Retail Fuel Availability
The retail fuel availability equation determines the share of stations offering the each fuel, $\sigma_{trc}{}^R$, from the fraction of capacity which is dedicated to the fuel of interest ($K_{trc}{}^R/K_{trc}$) and the endogenous conversion share, θ_{trc}, for each station which offers the fuel.

$$\sigma^R_{trc} = \frac{K^R_{trc}}{K^R_{tr}\theta_{trc}} \quad \forall \quad t,r,c \in C_{RET} \tag{62}$$

Equation of Motion for Installed Retail Capacity
As with all non-vintaged, durable capital, retail capacity in year t+1 is equal to the surviving retail capacity at the end of year t plus any new investments. Retail capacity is tracked by fuel type and year.

$$K_{t+1,f} = K_{tf}(1-\delta) + I_{tf}, \quad \forall \quad t+1,f \tag{63}$$

Retail Fuel Path Smoothing Equations

The retail fuel path equations help avoid radical variations in the modeled fuel retailing behavior. The model chooses both the total capacity for each fuel and the degree to which that capacity is concentrated in a few stations or spread over many stations. The average fraction of a station's fuel pumps which are dedicated to a given fuel type at each station that offers the fuel is θ_{trc} and these constraints limits its variation to 15% per year:

$$\begin{aligned} \theta_{t+1,rc} &\geq 0.85\,\theta_{trc} \quad \forall c \;\varepsilon\; C_{RET} \\ \theta_{t+1,rc} &\leq 1.15\,\theta_{trc} \quad \forall c \;\varepsilon\; C_{RET} \end{aligned} \tag{64}$$

7.0 WHOLESALE FUEL PRODUCTION AND TRANSPORTATION

The production functions of the various vehicle fuels at the wholesale level differ in their approach depending on the fuel type. The wholesale production of gasoline, natural gas, CNG and LPG are based on mature technologies and are represented by variable elasticity supply curves. On the other hand, the price of methanol can be expected to substantially change with the scale of production. Wholesale methanol production, therefore, explicitly tracks the level of investment in methanol production capacity. The cost of methanol production also varies with the price of its feedstock methanol. The wholesale price of ethanol is strongly dependent on assumption about the rate of progress in ethanol production technology and is expected to decline over time. Electricity costs are set at the rate for industrial customers. The production technologies used with each fuel are discussed below.

7.1 Fuel Feedstock Supply for Natural Gas, Ethanol, and Gasoline

This module is straightforward. Given supply (marginal cost curves) for gasoline, natural gas and ethanol feedstocks (grain and cellulosic), a cost function associated with the supply of each is easily constructed. The functional forms for supply curves are those used in AFTM, although constant elasticity forms could also be used.

The feedstock supply parameters and data drawn from ORNL (Walsh, et al., 1997), EIA (1996a) and Leiby (1993). In order to facilitate smooth supply paths and standardize across feedstocks, aggregate feedstock supply curves derived from these sources were fitted to variable elastic supply curves using the inverse form:

$$P_i = A + \frac{B}{C - Q_i} \tag{65}$$

For each feedstock, variables A, B, and C were chosen optimally in order to minimize the mean squared error between the original curves and the fitted curves. Given this functional form, nonlinear and near-linear supply curves can then be fully characterized by three intervals: (P1, 0), the price at which quantity supplied is zero, (∞, Q3), the quantity at which price is infinite, and any interval along the supply curve (P2, Q2). Table 16 below shows the 1995 and 2010 supply parameters for use in the TAFV. Supply curves for years not specified below are interpolated by the model.

Table 16: Fuel Supply Parameters for TAFV ($94, millions bbl/day)						
1995 Values						
Fuel	**P1**	**Q1**	**P2**	**Q2**	**P3**	**Q3**
Natural Gas (bbl FOE)	10.001	0.000	11.505	1.687	∞	10.592
Gasoline (physical bbl)	22.495	0.000	29.123	7.038	∞	238.013
Moderate Biomass Ethanol (physical bbl)	47.770	0.000	56.545	0.500	∞	1.500
Moderate Corn Ethanol (physical bbl)	42.896	0.000	48.745	0.130	∞	4.200
Optimistic Biomass Ethanol (physical bbl)	47.770	0.000	56.545	0.500	∞	1.500
Optimistic Corn Ethanol (physical bbl)	38.021	0.000	44.358	0.130	∞	5.100
2010 Values						
Natural Gas (bbl FOE)	13.399	0.000	14.903	1.687	∞	10.592
Gasoline (physical bbl)	31.723	0.000	39.184	7.893	∞	238.013
Moderate Biomass Ethanol (physical bbl)	30.025	0.000	34.155	1.168	∞	2.634
Moderate Corn Ethanol (physical bbl)	41.954	0.000	47.955	0.130	∞	4.944
Optimistic Biomass Ethanol (physical bbl)	28.512	0.000	31.919	1.435	∞	3.326
Optimistic Corn	35.016	0.000	41.986	0.130	∞	4.837

7.1.1 Gasoline

Wholesale gasoline supply curves are estimated from the AFTM model. Scaling the reference demand for gasoline vehicles in the AFTM base case from 20 to 160 percent of the reference quantity generates multiple equilibrium price and quantity values. Since this results in shifts in the demand curves and movements along the supply curves, this approach reveals the underlying AFTM supply curves for conventional and reformulated gasoline. The two gasoline types are then combined to produce an aggregate 2010 gasoline supply curve and fitted to the variable elastic supply curve form as described above. In order to replicate 1995 and 2010 AEO 1996 values, the supply curve is shifted up or down, thus maintaining its basic form while reproducing AEO baseline estimates.[15] This approach assumes the basic shape of the 2010 AFTM supply curve also applies in 1995.

7.1.2 Natural Gas

Like the gasoline supply curves described above, natural gas supply curves for 1995 and 2010 are derived from the AFTM base case. Varying the reference demand for CNG vehicles from 0 to 160 percent of the base reference demand level produces points along the 2010 supply curve for natural gas. Subtracting boiler and retail natural gas quantities and using the prevailing wellhead natural gas price produces the net natural gas supply curve to motor vehicles. The resulting net natural gas supply curve is then fitted to the variable elastic supply curve form described above. Like the gasoline supply curve, the resulting fitted curve was benchmarked to the AEO natural gas-to-vehicles quantities and prevailing wellhead prices in 1995 and 2010.

7.1.3 Ethanol

Ethanol price and quantity values for biomass and corn aggregate supply curves are derived using the separate least cost aggregation across viable feedstocks. Biomass feedstocks include switchgrass, short rotation woody crops, agricultural residues, refuse-derived fuel, hardwoods, and softwoods. Corn feedstocks include corn used in new wet mill facilities, add-on wet mill facilities, and new dry mill facilities. Feedstock supplies are characterized by intercepts, slopes, and transportation and conversion costs as well as availability constraints. These were provided by the ORNL/DOE ethanol transition study.[16] Aggregate supply curves are generated through minimizing the total cost of achieving each aggregate supply requirements from 0.1 billion gallons of ethanol up to the availability limit in increments of 100 million gallons based upon the availability and costs of the feedstocks. The minimization problem for both aggregate feedstocks is given below.

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[15] Wholesale conventional gasoline price data was provided by Stacy McIntyre at the EIA. Light-duty vehicle (less motorcycles) quantities were derived from The Supplement to the Annual Energy Outlook, 1996, Section D, Tables 32, 34 and converted to millions of barrels per day from trillions of BTU's per year using the appropriate method.

[16] The authors wish to thank Marie Walsh for her assistance in this area.

$$\underset{\{q_i\}}{Min} \quad TC = \sum_{n=0}^{N} \sum_{f=1}^{F} [\int_{0}^{\bar{q}} C_f(\alpha_f + \beta_f C_f q_{nf})\, dq \; + \; C_f T_f q_{nf} \; + \; PG_f q_{nf}]$$

s.t.

$$\text{feedstock limit contraint:} \qquad q_{nf} \;\le\; \bar{q}_f \qquad (66)$$

$$\text{aggregate supply constraint:} \quad \sum_{f=1}^{F} q_{nf} \;=\; Q_n$$

$$\text{nonnegativity constraint:} \qquad q_{nf} \;\ge\; 0$$

where:
q_{nf} = quantity of feedstock *f* (ethanol gallons/year) used in aggregate quantity iteration *n*
C_f = conversion rate (dry tons/ethanol gallon)
α_f = feedstock intercept ($/dry ton)
β_f = feedstock slope ($/dry ton)
T_f = transportation cost ($/dry ton)
PG_f = plant level conversion cost ($/ethanol gallon)
$\bar{q}_f$ = feedstock quantity limit (ethanol gallons/year)
Q_n = aggregate quantity (ethanol gallon/year) stipulated for quantity iteration *n*.

The resulting Lagrangian and partials are:

$$\pounds \;=\; \sum_{n=0}^{N} \sum_{f=1}^{F} [\int_{0}^{\bar{q}} C_f(\alpha_f + \beta_f C_f q_{nf})\, dq + C_f T_f q_{nf} + PG_f q_{nf} - \lambda_f(q_{nf} - \bar{q}_f) - \varphi_f(\sum_{f=1}^{F} q_{nf} - Q_n) - \delta_f(q_{nf})]$$

$$\frac{\partial \pounds}{\partial q_{nf}} \;=\; \begin{pmatrix} \textit{Plant gate price} \\ \textit{of feedstock } f \end{pmatrix} \;=\; \varphi_f \;=\; C_f(\alpha_f + \beta_f C_f q_{nf}) + C_f T_f + PG_f - \lambda_f - \delta_f \qquad (67)$$

The prevailing price of either biomass or corn derived ethanol for any quantity supply requirement is then the accompanying shadow value of an additional unit of feedstock ($/gal). The plant gate price of any feedstock is equal to the marginal cost of feedstock supply (feedstock, transportation, and conversion costs) less the shadow values of the nonnegativity and feedstock limit constraints. The plant gate price is equal to the marginal cost of feedstock supply if and only if $0 < q_{nf} < \bar{q}_{nf}$. which implies that a nonnegative feedstock quantity is optimal and desired, and that a feedstock quantity is not at its limit.

The resulting ethanol supply curves are then fitted to the variable elastic form described above.[17]

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[17] Further documentation can be found in *Methodology for Constructing Aggregate Ethanol supply Curves* (1997) and *Evolution of the Fuel Ethanol Industry: Feedstock Availability and Price* (1997).

The 1995 supply curve was extrapolated back from the generated 2000 -2015 supply curves.

7.2 Methanol

Wholesale methanol production costs are derived from a DOE study (U.S. DOE, 1989). Cost are representative of a standard 2,500 MTPD and an advanced 10,000 MTPD plant producing fuel grade methanol. Consistent with the TAFV model's approach to requiring sunk investment to produce service flows, methanol production costs are broken into variable and sunk components. Variable cost components, that is costs that are only incurred on a per-unit basis when used, include natural gas and non-gas operating costs. The cost of natural gas is determined endogenously within the model via the natural gas supply function. Sunk costs include the cost per unit of methanol capacity and are borne regardless of the level of production. This feature allows for the existence of costly, excess methanol capacity.

The natural gas supply function, non-gas operating and capital costs assume domestic gas production and costs of capital and construction. In particular, methanol production is assumed to occur at "category 1" sites that are already developed with access roads, readily available electric power supply and other industrial infrastructure in place (U.S. DOE, 1989, p. 5). Natural gas costs at category 1 are relatively expensive. Non-gas operating and sunk capital cost for the 2,500 and 10,000 MTPD are shown in Table 17.

Table 17: Wholesale Methanol Production Cost Parameters (1994 $)

	Capital	Capital	Non-Gas Operating	Non-Gas Operating	Output
	Million $	$ per Barrel	Million $ Per Year	$ per Barrel	Million Barrels Per Year*
Site 1 - Standard Scheme, 2,500 MTPD	$303.52	$45.89	$29.95	$4.53	6.61
Site 1 - Advanced Scheme, 10,000 MTPD	$782.32	$29.57	$84.07	$3.18	26.46

Source: Derived from USDOE 1989.

*Output per year assumes 8000 hours of on-stream time.

7.2.1 Implied Annual Capital Charge for Methanol Capital Stock

Methanol production facilities are treated as unvintaged capital stock. The cost of unvintaged capital is imposed as a lump-sum initial charge at the time of investment. Due to the depreciation of capital and the discounting of future use benefits, there is an implied capital charge that is borne by each unit of production.

In the TAFV model, financial flows are discounted with an annual discount factor $\delta = 1/(1+r)$, and each year the fraction γ of the unvintaged capital stock is scrapped. In a steady-state equilibrium, with capital stock fully utilized, the net present value of implied annual capital charges $\underline{C}_t^K$ will equal the original investment cost C_0:

$$C_0 = \sum_{\tau=0}^{\infty} \underline{C}_\tau^K \delta^\tau (1-\gamma)^\tau \tag{69}$$

The capital charge *rate* r_K is the ratio of the annual capital charge to the initial investment cost:

$$\begin{aligned} r_K &\equiv \frac{\underline{C}^K}{C_0} \\ &= \frac{1}{\sum_{\tau=0}^{\infty} \delta^\tau (1-\gamma)^\tau} = \frac{1}{\sum_{\tau=0}^{\infty} \left(\frac{1-\gamma}{1+r} \right)^\tau} \end{aligned} \tag{70}$$

(This result is shown in the appendix for the case of an infinite lived vintaged stock with a constant scrappage rate). Solving for the infinite sum in the denominator yields:

$$\begin{aligned} \sum_{\tau=0}^{\infty} \left(\frac{1-\gamma}{1+r} \right)^\tau &= \frac{1}{1 - \frac{1-\gamma}{1+r}} = \frac{1+r}{r+\gamma} \\ r_K &= \frac{r+\gamma}{1+r} \end{aligned} \tag{71}$$

Using this relationship, we can choose to a scrappage rate such that the capital charge rate used by in the US DOE (1989) report is consistent with the charge per-unit of capital used in the TAFV model. Solving the above equation for the scrappage rate yields.

$$\gamma = (1+r) r_K - r. \tag{72}$$

In the USDOE report the capital charge rate is 20%, and in the TAFV model, all costs and benefits are discounted at a 10% discount rate. This means that the scappage rate, γ, should be set to be 12%.

7.3 Fuel Transportation and Distribution Markup Costs

As is shown in Table 18 below, the cost of transportation and distribution for fuels used in TAFV. The estimates are in barrels of gasoline equivalents and account for all transportation and distribution costs from the production facility (plant gate) to the retail outlet. In addition, LPG and CNG include the cost of conversion from natural gas.

Table 18: Transportation, and Distribution Costs ($94/BGE)		
Year	1995	2010
Gasoline	1.54	154
E85	2.35	2.35
M85	2.95	2.95
LPG	22.30	28.17
CNG	7.59	7.59

Transportation and distribution costs for ethanol and gasoline are from the *Assessment of Ethanol Infrastructure for Transportation Use*, NREL, 1991. These include rail, pipeline, barge, and truck costs. In this report, the cost of E85 is calculated assuming equal densities between ethanol and gasoline and slightly higher transportation costs on a physical gallon basis. Methanol costs are estimated by applying ethanol costs on a physical gallon basis to methanol. The lower BTU content of methanol relative to ethanol results in higher M85 costs on a BGE basis.

LPG transportation and distribution markups are calculated as the difference between the wellhead price of natural gas and the industrial price of LPG as given in the AEO96. It is assumed that the cost of converting natural gas to LPG is imbedded in these costs. The transportation and distribution costs of CNG are based upon the historic difference between CNG citygate and LPG wellhead (Natural Gas Annual, 1995) and also include the cost of conversion.

7.4 Conversion and Efficiency Assumptions

Alcohol FFV vehicles are assumed to gain 1% in efficiency when using alt fuel. This is reflected in conversion of alt fuel (i.e. E85 & M85) to aggregate FFV fuel (e.g. MTHG).

Alcohol use in Dedicated AFVs assumed to gain 5% in efficiency (See B. McNutt memo to B. Massell, DOE, 8/13/93).

All final transportation fuels are converted to BGE based on their lower heating value.

8.0 MODEL VARIABLES AND SOLUTION METHODOLOGY IN DETAIL

The section presents the variables and equations used in the TAFV model version 1.0. As described above, the TAFV model is build around the concept of a competitive market for transportation services. A competitive equilibrium occurs when private marginal consumption benefits equal private marginal production costs (i.e., those quantities where supply curves and demand curves intersect). The TAFV model equate the demand for transportation services by private consumers and fleet owners with the supply of transportation services available at different prices. Price responsive supply and demand functions, conversion processes, and nested multinomial choice behavior are represented.

8.1 Summary of TAFV Model Notation

Define the following subscripts or indices:

t indexes Time periods

- t_0 First historical period
- t_i Initial forecast period
- T Last period

F Set of commodities (fuels and vehicles, etc.), which are produced or consumed.

- f Indexes commodities, $f \varepsilon F$
- F_{NMF} Subset of commodities including Non-Motor Fuel commodities, $F_{NMF} \subseteq F$
- F_{MF} Subset of commodities including Motor Fuel commodities, $F_{MF} \subseteq F$
- F_{MV} Subset of commodities including Motor Vehicle services, $F_{MV} \subseteq F$
- F_{SW} Subset of commodities including composite (aggregate) commodities, which are composed of a variety of SWitchable (substitutable) inputs, $F_{SW} \subseteq F$

C Set of linear (fixed input-output coefficient) conversion processes

- c Indexes Conversion processes, $c \varepsilon C$
- C_{SW} Subset of processes tracking SWITCHABLE inputs to commodity choice function, $C_{SW} \subseteq C$
- C_{MV} Subset of processes producing Motor Vehicle services, $C_{MV} \subseteq C$
- C_{DUR} Subset of conversion processes using DURable capital stock, $C_{DUR} \subseteq C$
- C_{DURV} Subset of DURable conversion processes using Vintaged capital stock, $C_{DURV} \subseteq C_{DUR}$
- C_{DURU} Subset of DURable conversion processes using Unvintaged capital stock, $C_{DURU} \subseteq C_{DUR}$
- C_{DURNEW} Subset of DURable conversion processes using New capital stock, $C_{DURNEW} \subseteq C$
- C_{VPROD} Subset of conversion processes for Vehicle PROduction, $C_{VPROD} \subseteq C_{DUR}$
- C_{RET} Subset of conversion processes for motor fuel RETail, $C_{RET} \subseteq$
- C_g Set of processes yielding substitutable inputs for composite (aggregate)

	commodity g (establishes correspondence between aggregate Switchable output good and conversion processes yielding substitutable inputs.) $C_g \subseteq C$ for all $g \; \varepsilon \; F_{SW}$.
$C_g^{\,1}$	Set indicating the conversion process that yields the first (numeraire) Switchable input for composite good g (usually the conventional vehicle or fuel). $C_g^1 \; \varepsilon C_{g.}$
R	Set of supply, demand regions
r,ρ	Index supply, demand regions
a	Indexes age of vintaged capital equipment $0 \leq a \leq$ A

8.2 Model Objective Function

The objective function is a measure of net national social surplus from transportation services. Absolute levels of the objective function do not have much meaning since it is not possible to know the total value of transportation services in the United States. Changes in the objective function, however, do present meaningful estimates of the net costs and benefits of potential policies relative to a base case. The objective function is composed of the discounted sum of the individual period contributions to net benefit (N_{tr}) for time periods t and regions r, plus the final period valuation of durable capital stock ($F_{Tr}^{\;K}$):

$$N(d,s,a,x) \equiv \sum_r \left(\sum_{t=0}^{T} \delta_t N_{tr} + F_{Tr}^{K} \right) \tag{73}$$

8.3 Single Period Net Benefit Function

The single period net benefit functions, for each time t and region r, is comprised of the benefits of consumption demand B^D minus the costs of the chain of activities necessary to provide consumption goods. Those cost include raw material supply costs C^M, conversion costs C^C, long-lived capital costs C^K, retailing costs C^R, transportation costs (T per unit), sharing or diversity costs C^S for substitutable fuels and vehicles, fuel availability costs C^A and the costs of limited vehicle diversity, $\mathbf{C^V}$. The effects of tax incentives, $\boldsymbol{\tau}$, are also included. Using the following notation,

B^D	Benefits from demand/consumption and costs from raw material supply
C^M	raw material supply costs
C^C	conversion costs
C^I	unvintaged durable capital investment costs
C^K	vintaged durable capital costs
C^R	retailing costs
T	transportation costs (T per unit),
C^S	sharing or diversity costs for substitutable goods (fuels and vehicles)
C^A	limited fuel availability costs
$\mathbf{C^{VD}}$	limited vehicle diversity costs
$\mathbf{C^{VP}}$	vehicle production plant capital costs

the single period net benefit measure is:

$$N_{tr} = \Bigg\{ \sum_f \Big[B^D_{trf}(d_{trf}) - C^M_{trf}(s_{trf})\Big] - \sum_c C^C_{trc} a_{rct}$$
$$- \sum_{c \in C_{DURV}} C^K_{trc}(I_{trc}, K_{trc}) - \sum_{c \in C_{DURU}} C^I_{trc}(I_{trc}) - \sum_{c \in C_{VPROD}} C^{VP}_{trc}(K_{trc}, n_{trc}) - \sum_{c \in C_{RET}} C^R_{trc}(K^R_{trc}, \theta_{trc}) \qquad (74)$$
$$- \sum_f \sum_\rho T_{fr\rho} x_{tfr\rho} - \sum_{g \in F_{SW}} C^S_{trg}(q^C_{trg}) - \sum_{c \in C_{RET}} C^A_{trc}(\sigma^R_{trc}) - \sum_{c \in C_{VPROD}} C^{VD}_{trc}(n_{trc}) - \sum_c \tau_{tr} a_{trc} \Bigg\}$$

Here the variables are:

d_{trf}, s_{trf}	demand and supply quantities for fuel f in region r at time t
a_{trc}	activity level for conversion process c
$x_{tf\rho r}$	shipment of fuel f from region ρ to r
q_{trg}^{c}	composite (substitutable) goods quantity for composite good g
I_{trc}	investment in durable equipment for process c
σ_{trc}^{R}	retail availability for fuel retail activity c
θ_{trc}	retail station pump share for fuel retail activity c, in region r

parameters or inputs:

A_{fc}	commodity f output (input) per unit process c
C_{rc}^{c}	process c unit conversion cost, in region r
$T_{f\rho r}$	unit cost of shipping commodity f from region ρ to r
θ_{trc}^{min}	retail station minimum pump share for fuel retail activity c, in region r
τ_{trc}	taxes
δ_t	discount factor in year t, i.e. $1/(1+r)^t$

Each of the above general functions can be further broken down into their individual components. In most cases these components are given below in the functional form as used in the computer code.

8.3.1 End-User Consumption Benefits

End-user consumption benefits are measured by consumer surplus, assuming an isoelastic demand curve. For simplicity, the quantity of transportation services demanded d(p) is assumed to have the following isoelastic form: $d(p) = d_0(p/p_o)^{-\varepsilon}$, where d_0 is the reference quantity of transportation services demanded at the reference price p_0 and ε is the price elasticity of demand (defined positively). Consumer surplus is calculated as the area under the inverse demand (or marginal benefit) curve $D_{trf}^{-1}(q)$, or using our functional form: $p = p_0(d/d_0)^{-\frac{1}{\varepsilon}}$. Integration yields the following expression for consumer surplus

$$B^D_{trf}(d_{trf}) = \int_0^{d_{trf}} p_0(d/d_0)^{-\frac{1}{\varepsilon}} = A^d_{trf} \cdot d_{trf}^{b^D_{trf}} - c^D_{trf} \qquad (75)$$

where $A^D_{tfr} = p_0(1/(\frac{-1}{\varepsilon}+1)) \cdot q_0^{(1/\varepsilon)}$, and $b^D_{tfr} = (-1/\varepsilon + 1)$, and c^D_{tfr} is a constant of integration.

8.3.2 Raw Materials Supply Costs

Raw material supply costs are measured by producer costs given by the integral under the inverse-supply (or marginal cost) curve S_{trf}^{-1}. The marginal cost curve used implies a variable-elasticity of supply:

$$C_{trf}^{M}(s_{trf}) = \int_{0}^{s_{trf}} S_{trf}^{-1}(q)dq = a_{trf}^{S}s_{trf} - b_{trf}^{S}\ln(c_{trf}^{S} - s_{trf}) \tag{76}$$

8.3.3 Conversion Variable Costs

Conversion variable costs are linear in the level of activity a_{trc}.

$$C_{trc}^{C}(a_{trc}) = \underline{C}_{trc}^{C} a_{trc} \tag{77}$$

8.3.4 Retail Mark-up Costs

Retail mark-up costs are explained in detail above.

$$C_{trc}^{R}(K_{trc},\theta_{trc}) = K_{trc}\underline{C}_{c}^{R}\left(\frac{\bar{\theta}_{c}}{\theta_{trc}}\right)^{\eta^{R}} \quad \forall\ c \in C_{RET} \tag{78}$$

Here $\underline{C}_c^R$ is the cost per unit retail capacity, when fuel capacity is installed at each station at the reference fraction of pumps, $\bar{\theta}_c$. When the installation fraction θ_{trc} is lower than the reference level, costs per unit retail capacity are higher according to the elasticity η^R. Retail motor-fuel markup is handled separately to account for possible underutilization of retail capacity for new fuels. Retail capacity K_{trc}^R is treated as durable or quasi-fixed, and evolves with new investment and scrappage like other unvintaged capital.

8.3.5 Transportation Costs

Transportation costs for each origin-destination pair (r,ρ) are the unit transportation costs $T_{fr\rho}$ times shipment quantity $x_{tfr\rho}$:

$$T_{fr\rho}\, x_{tfr\rho}$$

8.3.6 Durable Vintaged Capital Stock (and Investment) Costs - Annual Capital Charges

Durable vintaged capital stock costs include an up-front costs at investment time, $\underline{C}_c^I$, and alternatively *may* include a unit-charge, $\underline{C}_c^K$, (called “STKCHARGE”) for each unit of investment and installed capital. No adjustment costs are imposed.

$$C_{trc}^{K}(I_{trc},K_{trc}) = I_{trc}\underline{C}_{c}^{I} + K_{trc}\underline{C}_{c}^{K} \quad \forall\ c \in C_{DURV} \tag{79}$$

8.3.7 Durable Unvintaged Capital Investment Costs

Durable unvintaged capital costs may be applied as a lump-sum charge, $\underline{C}_c^{\,I}$, at the time of investment. An adjustment cost factor may also be imposed to increase the unit investment cost at higher rates of investment, by specifying a non-zero value for C_c^{A}. Adjustment costs increase as investment approaches the specified maximum investment quantity, $\underline{I}_{trc}^{max}$. The general form is given below.

$$C_{trc}^{I}(I_{trc}) \;=\; I_{trc}\underline{C}_c^{I}\left[1+C_c^{A}\frac{I_{trc}}{(I_c^{\max}-I_{trc})}\right] \quad \forall\ c \ \in \ C_{DURU} \tag{80}$$

8.3.8 Vehicle Production Plant Costs (Total Vehicle-Type Specific Durable Capital Costs)

Total incremental capital costs for vehicle production capacity are calculated here. Since marginal capital costs depend upon the scale of each vehicle production *plant*, the incremental capital cost calculation is a function of total industry capacity (K_{trc}) and the number of vehicle models/plants, N_{trc}. The minimum incremental marginal capital costs, P_c^{min}, occurs for very large-scale production, and marginal capital costs are greater for lower scale (lower K/N) according to the parameter B_c^{V}.

$$\begin{aligned} C_{trc}^{VP} \;=\; N_{trc}\Bigg[&\left(P_c^{\min}\frac{K_{trc}}{1.15\kappa_c N_{trc}} \;+\; B_c^{V}\ln\left(\frac{K_{trc}}{1.15\kappa_c N_{trc}}\right)\right) \\ &-\left(P_c^{\min}\frac{\underline{K}_c^{\min}}{1.15\kappa_c}+B_c^{K}\ln\left(\frac{\underline{K}_c^{\min}}{1.15\kappa_c}\right)\right)+P_c^{NUMMOD}\Bigg]\frac{1}{1000} \quad \forall\ c \in C_{VPROD} \end{aligned} \tag{81}$$

These are the incremental costs of plant capital specific to vehicle type *c*. Variable and non-incremental (generic) capital costs are captured in the conversion cost terms. Note that total vehicle production stock *K* is divided by 1.15 to convert from maximum production capacity to rated capacity, since the cost functions are specified in terms of rated capacity. $\underline{K}_c^{min}$ is the minimum *plant* capacity for vehicle type *c* (which equals the minimum industry capacity since the minimum number of plants is one).

The division by 1000 is needed because vehicle capacity and production are measured in millions of vehicles, and costs are measures in billions of dollars. Non-incremental variable and capital vehicle production costs are represented as conversion variable costs (for the conversion processes associated with vehicle production).

8.3.9 Utilization and Sharing Costs (For Vehicle and Fuel Choice)

“Sharing costs” may occur for all composite commodities *g*, in order to account for the value that consumers give to a diverse mix of input alternatives. This value derives from the underlying random-utility choice model and the accounting for non-price attributes in each consumer’s discrete choice between vehicles and fuels. The form used for such sharing costs is given below.

$$C_{tr}^{S} = \sum_{g \in F_{sw}} C_{trg}^{S}$$

$$\begin{aligned} C_{trg}^{S}(q_{trg}^{c}, \vec{s}_{trg}) &= q_{trg}^{c}\left(-\frac{1}{\beta_g}\right)\sum_{f \in F_g}\left[\ln\left(\frac{s_{trf}}{s_{trf}^{*}}\right)\right]s_{trf} \quad \text{for } s_{trf}^{*} \text{ equal price shares} \\ &= q_{trg}^{c}\left(-\frac{1}{\beta_g}\right)\sum_{c \in I_g}\left[\ln\left(\frac{a_{trc}/q_{trg}^{c}}{s_{trgc}^{*}}\right)\right]\frac{a_{trc}}{q_{trg}^{c}} \quad \forall\ g \in F_{SW} \end{aligned} \tag{82}$$

8.3.10 Retail Fuel Availability Cost

Retail fuel availability cost to consumers represents the effective cost when motor fuels are not available at all stations (reflects expected inconvenience and incremental travel effort). Retail fuel availability is an endogenous non-price fuel attribute)

$$C_{trc}^{A}(\sigma_{trc}^{R}, a_{trc}) = \left(C_{trc}^{Aref} e^{b(\sigma_{trc} - \sigma_{trc}^{ref})} - C_{trc}^{Aref}\right)\frac{Q_{gaso}^{R}}{Q_{C}^{R}} a_{trc} \quad \text{for } c \text{ a retail process } (c \in C_{RET}) \tag{83}$$

These costs are linear in the amount of each fuel retailed (a_{trc}, the level of activities of the retail process). Here C^{Aref} is the cost per gallon at the reference availability σ^{ref}, and b is the elasticity of availability cost with respect to availability share. Subtracting C^{Aref} assures that the incremental availability cost is zero at complete availability (σ=1). The relative frequency of refilling (due to differing energy density of the fuel and possible differing range) is reflected by the ratio of gallons per gasoline refill to the number of gallons of gasoline equivalent per refill with fuel *c*, Q^{R}_{gaso}/Q^{R}_{c}. This equation is consistent with a retail fuel availability unit cost of the form $U(0)/\beta\ e^{\sigma\varepsilon}$.

8.3.11 Limited Vehicle Model Diversity Costs

The choice among vehicles types is also influenced by the diversity of vehicle models offered in each fuel-type/size class. Model diversity is measured relative to conventional gasoline vehicles. The effective cost of limited diversity is a function of the relative number of models, N_{trc}, produced for each fuel type, c_{tic}, compared to the reference number of models offered in the conventional (gasoline) type, N_{trg}. This effective cost of diversity is another component of the consumer's valuation of the non-price attributes for each vehicle type. These costs of limited diversity are treated as added production costs for *new* vehicles. They correspond to endogenous changes in the non-price attributes of vehicles in the multinomial logit choice.

$$C_{trc}^{VD}(N_{trc}, a_{trc}) = \left(\frac{\omega}{\kappa_g \beta_{trg}}\ln\left(\frac{N_{trc}}{N_{trg}}\right)\right)a_{trc} \quad \forall\ c \in C_{VPROD} \tag{84}$$

We know that $\ln(N_{trc}/N_{trg})$ is the utility change associated with vehicle diversity. We want to convert (84) to dollars, so we need the correct β (the marginal utility of dollar). The β for vehicle services choice is measured in units of marginal utility per dollar of vehicle services (BGSE), this marginal utility needs to be adjusted by the stock-per-flow parameter, κ_c, to convert vehicle services to vehicles. After making this adjustment to β yields the right units for effective cost in dollars per vehicle, not dollars per BGSE. The marginal utility of dollars term β is in utils per 1000 per vehicle, hence unit costs are in \$1000 per vehicle. Since vehicles are measured in units of millions per year, total costs are in billions of dollars per year.

8.3.12 Taxes and Financial Incentive

Taxes and subsidies are associated with conversion processes. The costs of taxes are a transfer, and will be rebated to country's net benefits in the *ex post* solution reports They are included because they affect the market outcome. Taxes are represented as positive values for τ, and subsidies are negative ($\tau_{trc} < 0$ promotes activity a_{trc}):

$$\sum_{t,r,c} \tau_{trc} a_{trc} \tag{85}$$

8.3.13 Final Value

The final value is estimated for accumulated vintaged and unvintaged capital in the terminal time period T. For unvintaged capital, the history of investments is scanned, and the appropriate scrappage rate for each year's investment is applied. The remaining unvintaged capital and the current investment in unvintaged capital are valued at their (exogenously estimated) future use value, $V_{Trc}{}^{U}$. The installed base of vintaged capital *and* the current investment expected to survive to the *end* of the last period is valued at the (exogenously estimated) future use value for capital of age a ($V_{Trac}{}^{V}$). Since the current use-value of vintaged and unvintaged capital during final period T is included elsewhere in the model, the final value accounts for use-value starting at the end of period T. Hence, one extra period of discounting is applied.

Future value for accumulated vintaged and unvintaged capital by age and type

$$F^{K}_{Tr} = \delta^{S}_{T} \sum_{c \varepsilon C_{DURV}} \left(\sum_{\tau=0}^{T} I_{\tau rc} \Gamma_{(T-\tau+1)c} V^{V}_{Tr(T-\tau)c} \right) + \delta^{S}_{T} \sum_{c \varepsilon C_{DURU}} \left(K_{Trc} V^{U}_{Trc} + I_{Trc} V^{U}_{Trc} \right) \tag{86}$$

where $\delta_T{}^S$ is the discount factor for the salvage period (one period after final period T), Γ_{ac} is the multiperiod stock survival rate to age a, and $V^{V}{}_{Tr(T-\tau)c}$ is the salvage or future value per unit of vintaged stock of age (T-τ). $V^{U}{}_{Trc}$ is the salvage value per unit of unvintaged capital stock.

8.3.14 Calculation of Future Use Value

Future use value for vintaged and unvintaged capital ($\mathbf{V_{trac}{}^{V}}$ and $\mathbf{V_{trc}{}^{U}}$) is calculated based on the cost of new investment and the discount and scrappage rates. This approach implies that the discounted stream of annual use values will, in equilibrium, equal capital costs.

8.4 Model Constraints

The objective function above is maximized subject to materials balance, capital accumulation, shipping and other constraints. These constraints are given below.

8.4.1 Supply-Demand Balance Constraint

The supply-demand constraints require that price-sensitive demand and non-price-sensitive demands are equal to the price-sensitive and non-price-sensitive supplies in each region plus net exports.[18]

$$d_{trf} \le s_{trf}^{exog} - d_{trf}^{exog} + s_{trf} + \sum_{c} A_{fc} a_{tcr} + \sum_{\rho} (x_{tf\rho r} - x_{tfr\rho}) \quad \forall\ f,r,t \tag{87}$$

8.4.2 Aggregate Output from Switching Processes

The aggregate output from switching processes is sum of conversion process activity levels for all Switchable inputs to composite good HSW (SWTOTE):

$$q_{trg}^{c} = \sum_{c \varepsilon C_g} a_{trc} \tag{88}$$

8.4.3 Maximum Share on Conventional Vehicles

The maximum share for conventional inputs into choice process is an optional constraint which can limit the choice of gasoline or conventional vehicles in region r for all periods t (SWMAXSHREQ):

$$\sum_{c \in C_g^1} a_{trc} \le \sigma_g^{\max} q_{trg}^{c} \tag{89}$$

8.4.4 Unvintaged Durable Stock Equation of Motion (STKUEQ)

The age of unvintaged capital stock is not tracked. Nonetheless, the level of unvintaged durable capital is determined by applying a single, constant scrappage rate for each capital type and adding and new investment I_{trc}.

$$K_{(t+1)rc} = K_{trc}(1-\gamma_c) + I_{trc} \quad \forall \quad 0 \le t < T,\ r,\ c \in C_{DURU} \tag{90}$$

8.4.5 Output Capacity Constraint for Unvintaged Capital (STKUTYPEEQ)

The total unvintaged stock type equation requires that there be enough equipment to satisfy durable equipment services demand.

$$a_{trc} \le K_{trc}/\kappa_c \quad \forall\ t,r,c \in C_{DURU} \tag{91}$$

where κ_c is the amount of capital stock required per unit of operational capacity.

8.4.6 Vintaged Stock Total Equation (STKVTOTEQ)

Some processes (e.g., those choosing fuel use for existing vehicles) can only use old vintaged capital. The vintaged stock total equation sums up all used stock in each year for each process.

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[18]Non-price-sensitive demand includes the level of fleet vehicle transportation services demand. This demand is not-price sensitive since it is set exogenously by policy.

$$\sum_{\tau=0}^{T} \alpha_{t\tau} I_{\tau rc} = K_{trc}^{tot} \quad \forall \quad t,r,c \in C_{DURV} \tag{92}$$

Where $\alpha_{t\tau}$ is the fraction of capital investment from year τ which is remaining and available in year t. (For $t \geq \tau$, $\alpha_{t\tau}$, is the stock survival rate to age t-τ, and $\alpha_{t\tau}$ =0 for $t<\tau$).

8.4.7 Use of New Investment in Vintaged Stocks (NEWUSEEQ)

For some processes (e.g. those associated with new vehicles) the use or activity level is equal to level of new investment. The set of processes requiring the use of new capital (vehicles) is called CDURNEW. For each of those processes, the type of new investment required is indicated in the set association "CDURNEWOLD.":

$$a_{trc'} = \sum_{c \varepsilon C_{DURNEWOLD}} \frac{I_{trc}}{\kappa_c} \quad \forall \; c \in C_{DURNEW} \tag{93}$$

where each process c' using vintaged durable capital is associated with a process c through the association $C_{DURNEWOLD}$. This constraint is necessary to assure that new vehicle services are tied to new vehicle purchases.

8.4.8 Output Capacity Constraint for Vintaged Capital (STKVTYPEEQ)

The maximum conversion activity level (output) from processes using vintaged capital must be less than or equal to the remaining amount of installed capital after accounting for declining use rates and decay.

$$a_{trc} \leq \sum_{\tau=0}^{T} \varphi_{t\tau} I_{\tau rc} / \kappa_c \quad \forall \; t,r,c \in C_{DURV} \tag{94}$$

The parameter $\varphi_{t\tau}$ is the effective amount of capital purchased in year τ that remains available in year t, accounting for scrappage and reduced use with age. κ_c is the amount of <u>new</u> capital stock required per unit of operational capacity.

8.4.9 Stock Growth Limit Equation (VPSTKLIMEQ)

Some processes (e.g. vehicle production capacity) have annual limits on their rate of expansion, r_K^{max}.

$$K_{t+1,rc} \leq (1 + r_{Kc}^{\max}) K_{trc} \quad \forall \quad c \in C_{VPROD} \tag{95}$$

8.4.10 Vehicle Production Plant Size Minimum Equation (VPSTKMINEQ)

The vehicle production plant size equations require that vehicle production plants (total industry capacity K_{trc} divided by number of models N_{trc}) be at least of a minimum size at a plant-level of analysis.

$$K_{trc} \geq \underline{K}_c^{\min} N_{trc} \quad \forall \; t,r,c \in C_{VPROD} \tag{96}$$

8.4.11 Vehicle Production Plant Downsizing Limit (VEHPLANTSZ)

The vehicle production plant downsizing equation constrains plant size reductions to the plant scrappage rates. This prevents vehicle manufacturers from subdividing plants to produce more models.

$$\frac{K_{t+1,rc}}{n_{t+1,rc}} \geq \frac{K_{trc}}{n_{trc}}(1-\gamma_c) \quad \forall\ r, c \varepsilon C_{VPROD}\ \ t \geq t_0 \tag{97}$$

8.4.12 Motor Fuel Retail Constraint (RETKFE)

Determine the capacity to retail each motor fuel type from the stock of the associated retail (durable) conversion process. In the interest of calculating retail station costs, we track a variable for retail station capacity $K_{trc}{}^R$ ("RETKF") distinctly from general durable stock capacity K_{trc} ("STKTOTAL"):

$$K_{trc}^{R} = K_{trc} \quad \forall \quad c \in C_{RET} \tag{98}$$

Note: Since retail station capacity is treated as a durable stock, the stock type equation (STKUTYPEEQ) assures that the quantity of each motor fuel retailed must be less than the current retail capacity. Retail capacity $K_{trc}{}^R$ is treated as durable but has a high rate of turnover. In theory, retail capacity *could* exceed retail quantity, in the case where the cost of excess retail capacity is less than the value of increased fuel availability.

$$q_{trc}^{R} \leq K_{trc}^{R} \quad \forall t, r, c \varepsilon C_{RET} \tag{99}$$

8.4.13 Total Retail Capacity (RETKTE)

The total retail capacity equation tracks the retail capacity for all fuels, so that retail availability for each fuel may be calculated.

$$K_{tr}^{R} = \sum_{c \varepsilon C_{RET}} K_{trc}^{R} \tag{100}$$

8.4.14 Retail Fuel Availability (RETSHAREE)

The retail fuel availability equation determines the share of stations offering the each fuel ($\sigma_{trc}{}^R$ or "RETSHARE") from the fraction of capacity which is dedicated to the fuel of interest ($K_{trc}{}^R/K_{trc}$) and the endogenous conversion share (θ_{trc} or "RETTHETA") for each station which offers the fuel.

$$\sigma_{trc}^{R} = \frac{K_{trc}^{R}}{K_{tr}^{R}\theta_{trc}} \quad \forall \quad t, r, c \in C_{RET} \tag{101}$$

8.4.15 Retail Fuel Path Smoothing Equations

The retail fuel path equations (RETSMOOTHL and RETSMOOTHU) help avoid radical variations in the modeled fuel retailing behavior. The model chooses both the total capacity for each fuel and the degree to which that capacity is concentrated in a few stations or spread over many stations. The average fraction of a station's fuel pumps which are dedicated to a given fuel type at each station that offers the fuel is θ_{trc} (variable RETTHETA), and these constraints limits its variation to 15% per year:

$$\begin{aligned} \theta_{t+1,rc} &\geq 0.85\theta_{trc} \quad \forall c \ \varepsilon \ C_{RET} \\ \theta_{t+1,rc} &\leq 1.15\theta_{trc} \quad \forall c \ \varepsilon \ C_{RET} \end{aligned} \tag{102}$$

8.5.16 Concise Statement of TAFV Model Equations

Model Optimization Problem: The Dynamic Objective Function (OBJ) measures multiperiod world net social surplus. This includes the discounted sum of the individual period contributions to net benefit (N_{tr}) plus the terminal period valuation of durable capital stock ($B_{Tr}{}^{K}$). The Dynamic solution approach find the perfect-foresight competitive equilibrium by maximizing the discounted multiperiod social surplus:

$$\max_{d,s,a,x,I} N(d,s,a,x,I) \equiv \max_{d,s,a,x,I} \sum_{r} \left(\sum_{t=0}^{T} \delta_t N_{tr} + F_{Tr}^{K} \right) \tag{103}$$

subject to the following constraints:

Supply-demand balance

$$d_{trf} \leq s_{trf}^{exog} + s_{trf} + \sum_{c} A_{fc} a_{tcr} + \sum_{\rho} (x_{tf\rho r} - x_{tr\rho f}) \quad \forall \quad f,r,t \tag{104}$$

Summing up of substitutable inputs for composite goods

$$q_{trg}^{c} = \sum_{c \varepsilon C_g} a_{trc} \quad \forall \quad g \in C_{SW} \tag{105}$$

Capital stock evolution

$$K_{(t+1)rac} = K_{trac}(1-\gamma_{ac}) + I_{trc} \quad \forall \quad 0 \leq t < T, \ r, \ a=0, \ c \in C_{DUR} \tag{106}$$

Total Stock Type Equation

$$a_{trc} \leq \sum_{a \geq 0}^{A} K_{trac}/\kappa_c \quad \forall \quad t,r,c \in C_{DUR} \tag{107}$$

Stock Accounting equation

$$\sum_{a \geq 0}^{A} K_{trac} = K_{trc}^{tot} \quad \forall \quad t,r,c \in C_{DUR} \tag{108}$$

Set maximum allowable initial vintaged capital stock, for all ages

$$\sum_{a \geq 0}^{A} K_{0rac} \leq K_{rc}^{INIT} \quad \forall \quad r,a,c \in C_{DUR} \tag{109}$$

Initial age distribution (minimum initial stock for each age) of vintaged capital equipment

$$K_{0rac} \geq K_{0rc}^{TOT} \psi_{ac} \tag{110}$$

Quantity of each motor fuel type retailed

$$q_{trc}^{R} = a_{trc} \quad \forall \quad c \in C_{RET} \tag{111}$$

Retail quantity constrained by retail capacity

$$q_{trc}^{R} \leq K_{trc}^{R} \quad \forall \quad t,r,c \in C_{RET} \tag{112}$$

Total retail capacity tracking

$$K_{tr}^{R} = \sum_{c \varepsilon C_{RET}} K_{trc}^{R} \tag{113}$$

Retail availability

$$\sigma_{trc}^{R} = \frac{K_{trc}^{R}}{K_{tr}^{R}\theta_{trc}} \quad \forall \quad t,r,c \in C_{RET} \tag{114}$$

Final valuation of terminal capital stock (Dynamic Model)

$$B_{Tr}^{K} = \delta_{T} \sum_{c \varepsilon C_{DUR}} \left(\sum_{a}^{A} K_{Trac} V_{Trac} + I_{Trc} V_{Tr1c} \right) \tag{115}$$

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APPENDIX 1: SYMBOLS AND NAMES IN GAMS CODE

Indices	Sets	Meaning
t	T	time period (year)
f	H	commodities (fuels, vehicles)
g	HSW	substitutable commodities
c	C	conversion processes
a	AGE	age of vintaged durable capital
r,ρ	I	regions

Variables

s_{trf}	SUPPLY(T,I,H)	supply quantities
d_{trf}	DEMAND(T,I,H)	demand quantities
K_{trc}	STKTOTAL(T,I,CDUR)	stock totals
K_{trac}	STKV(T,I,AGE,CDURV)	vintaged stock
I_{trc}	INV(T,I,CDUR)	investment
a_{trc}	CONV(T,I,CDUR)	conversion levels
σ_{trc}^{R}	RETSHARE(T,I,CRET)	retail fuel availability
K_{trc}^{R}	RETKF(T,I,CRET)	total retail fuel capacity for fuel f
K_{tr}^{R}	RETKT(T,I)	total retail fuel capacity
θ_{trc}	RETTHETA(T,I,CDUR)	fraction retail capacity dedicated to fuel f
n_{trc}	NUMMOD(CURRT,I,CVPROD)	number of models

Parameters

κ_c	STKDATA("STKPERFLOW",CDUR)
$\theta_{c,ref}$	RETDATA("RETPUMPREF",CRET)
C_c^{R}	RETDATA("RETPUMPCST",CRET)
η^{R}	RETPUMPELS
ψ_{ac}	STKAGEPCTi
A_{fc}	CPRO(H,C)
$T_{fr\rho}$	UTC(H,I,J)
$x_{tfr\rho}$	SHIP(T,H,I,J)
q_{trg}^{c}	SWOUTPUT(T,I,HSW)
γ_{ac}	STKVSCRAPR(AGE,C)
γ_c	STKUSCRAPR(C)
$\underline{C}_{trc}^{C}$	CCST(T,I,C)
a_{trf}^{D}	DEMA(T,I,H)
b_{trf}^{D}	DEMB(T,I,H)
c_{trf}^{D}	DEMC(T,I,H)

a_{trf}^{S}	SUPA(T,I,H)
b_{trf}^{S}	SUPB(T,I,H)
c_{trf}^{S}	SUPC(T,I,H)

Parameters

$\underline{C}_c^{K}$	STKDATA("STKCHARGE",CDURV)	annualized charge per unit capital
$\underline{C}_c^{I}$	STKDATA("STKCOST",CDURU)	up-front capital cost per unit investment
$\underline{C}_c^{A}$	STKDATA("STKADJCST",CDURU)	adjustment cost factor for unvintaged stock only
I_c^{max}	STKDATA("STKADJMAX",CDURU)	max new investment rate
P_c^{min}	VPDATA("PMIN",CVPROD)	minimum incremental capital costs for vehicle production
B_c^{V}	VPDATA("PSLOPE",CVPROD)	
L	RETTOURLEN	
σ_{trc}	RETSHARE(CURRT,I,CRET)	
ρ	RETDENSITY	
$\underline{c}^{T}$	RETTRAVCST	
Q_c^{R}	RETFUELDAT("REF-FILLQ",CRET)	

Parameters

V^{V}_{trac}	FUTUREVV(T,I,AGE,C)	Final value per unit vintaged capital of age a
V^{V}_{tr1c}	FUTUREVV(T,I,"1",C)	Final value per unit vintaged capital of age 1
V^{U}_{trc}	FUTUREVU(T,I,C)	Final value per unit unvintaged capital
$\varphi_{t\tau}$	STKVUSER (CURRT,T,CDURV)	Effective number of vehicles purchased in year t which survive to year t (accounting for scrappage and declining usage with age).
r_{Kc}	STKDATA ("EXPRATE",CVPROD)	Max expansion rate for vehicle production capacity
$\alpha_{t\tau}$	STKAVAILR	Fraction of capital investment from year τ which is remaining and available in year *t*
$\underline{K}_c^{min}$	STKDATA ("STKMIN",CVPROD)	Minimum vehicle production plant size
δ_t	DISCFACT(T)	Period t discount factor
δ_T^S	DISCSALV(TLAST)	Discount factor for salvage period (final T+1)
Γac	STKVSURVIR (AGE,CDURV)	Multiperiod stock survival rate to age a
ω	CVEHDIVADJ	Parameter rescaling vehicle model diversity costs
β_{trg}	SWBETA	Marginal utility of $(income) in MNL choice function for good g

APPENDIX 2: TREATMENT OF VINTAGED CAPITAL

STOCK

This appendix develops the theory and practical equations necessary for properly treating the salvage value of vintaged capital stock in the terminal period. It provides background for the briefer discussion of these topics in Chapter 5.

A2.1 Vehicle Stock Shadow Value and Final Value

Since the period of analysis in TAFV is finite (years 1995-2010), care must be exercised to properly treat the final period. In a dynamic model some decisions have long-lasting effects. In particular, investments in durable capital influence capital stocks, prices, and profits for many subsequent years. The dynamic model seeks to characterize the competitive decisions by firms and consumers in making capital investments, which will naturally depend on the expected revenue to be gained from that capital in subsequent years. As the model analysis loops over years and approaches the final year in the finite horizon, some fraction of newly purchased durable capital will be expected to survive past the terminal time. The problem is to specify the "terminal value" or "final value" of surviving capital in the terminal period, so that the resulting private investment decisions are reasonable and follow a smooth path in the immediately preceding years.

For vintaged capital stock such as vehicles, the terminal value problem is somewhat complicated by the fact that each vehicle age will have a different final value, depending on its remaining life and utilization. The following sections describe the theory and method used to establish the final value of vehicle stocks.

A2.1.1 Problem Statement and First Order Conditions

The value of any capital, vintaged or unvintaged, stems from the anticipated net present value of its current and future usage. The net present usage value is the discounted value of the stream of future benefits (or revenues, for a firm) minus the usage costs (e.g. fuel or other variable inputs). In a competitive equilibrium, private agents will purchase additional capital until its net present usage value equals its cost. This can be shown by looking at a compact version of the equations of relevance from the TAFV model. Suppose consumers or firms derive the following costs and benefits from the purchase and use of vehicles:

- $B(q_t)$ benefits from the use of vintaged capital in time t;
- C variable costs of the use of capital (vehicles);
- C^I up-front purchase costs of capital (vehicles);
- C^S effective costs to consumers of vehicle mix due to non-price vehicle attributes.

The net benefits from capital are:

$$N(t,\vec{K}_t) = B(q_D) - C(a_U,a_N,\vec{K}_t) - C^I(I) - C^S(a_N)$$

The objective is to maximize the sum of discounted net value for all periods plus the terminal value of capital stock

$$Max \sum_{t=0}^{T} N(t,\vec{K}_t)\delta_t + \sum_{a} F_{Ta}(K_{Ta})\delta_T \tag{117}$$

with the constraints

s.t.

Variable Name:	*Equation*:	*L−mult Symbol*:	*Condition*:
STKVEQ:	$K_{t+1,a+1} = K_{ta}(1-\gamma_a)$	$\lambda_{Kt+1,a+1}$	$t=0......T-1,\ a=0......A-1$
NEWSTKEQ:	$K_{t0} = I_t$	λ_{Kt0}	$\forall t$
STKVTYPEEQ:	$a_{Ut} \le \sum_{a=1}^{A} \frac{K_{ta}}{\kappa_a} \equiv f_U(\vec{K})$	λ_{fUt}	$\forall t$
NEWUSEEQ:	$a_{Nt} = \frac{I_t}{\kappa_0} \equiv f_N(I_t)$	λ_{fNt}	$\forall t$
STKUTOTEQ:	$\sum_{a=1}^{A} K_{ta} = K_t$		$\forall t$
SDEQ:	$q_D \le a_U + a_N$	λ_{Bt}	$\forall t$
	$I_t \ge 0$	μ_{I_t}	$\forall t$
	$a_{Ut}, a_{Nt} \ge 0$	μ_{Ut}, μ_{Nt}	$\forall t$
STKPCTE0:	$K_{0a} \ge \alpha_a K_0$	λ_{K0a}	$a>0,\ t=0$

(118)

The appropriate Lagrangian may be written as the sum of "static" Lagrangians for each period plus the final value term, the stock dynamic constraint terms, and the initial stock constraint terms:

$$\begin{aligned} L = \sum_{t=0}^{T} L_s(t) + \sum_{a=0}^{A} F_{Ta}(K_{Ta})\delta_T + \sum_{t=0}^{T-1}\sum_{a=0}^{A-1} \lambda_{K,t+1,a+1}(K_{t+1\,a+1} - K_{ta}(1-\gamma_a)) \\ + \sum_{a=1}^{A} \lambda_{K0a}(K_{0a} - \alpha_a K_0) \end{aligned} \tag{119}$$

The single period "static" Lagrangians consist of the single period net benefit plus the static constraint terms which apply in every period:

$$
\begin{aligned}
L_s(t) \; = \;\; & N_S\delta_t \; + \; \lambda_{Kt0}(K_{t0} - I_t) \\
& + \; \lambda_{fUt}(a_{Ut} - \sum_{a=1}^{A} \frac{K_{ta}}{\kappa_a}) \\
& + \; \lambda_{fNt}(a_{Nt} - \frac{I_t}{\kappa_0}) \\
& + \; \lambda_{Bt}(q_D - (a_U + a_N)) \\
& + \; I_t\mu_{It} \\
& + \; a_{Ut}\mu_{Ut} + a_{Nt}\mu_{Nt}
\end{aligned} \tag{120}
$$

$$
N_s(t) \; = \; B_t(q_{Dt}) \; - \; C_t(q_D, a_U, a_N, \vec{K}_t) \; - \; C_{It}(I_t) \; - \; C_t^{\,S}(a_N) \tag{121}
$$

First look at the derivatives of static (single period) Lagrangian L_s's. These are the "static optimality" conditions, which apply to current period supply, demand, and conversion activities given current capital stock.

$$
\begin{aligned}
\frac{\partial L}{\partial I} \; = \; \frac{\partial L_s(t)}{\partial I_t} \quad & = \; -\frac{\partial C_{it}}{\partial I_t}\delta_t \; - \; \lambda_{Kt0} \; - \; \lambda_{fNt}\frac{\partial f_{Nt}}{\partial I_t} \; + \; \mu_{It} \\
& = \; -\frac{\partial C_{it}}{\partial I_t}\delta_t \; - \; \lambda_{Kt0} \; - \; \lambda_{fNt}\frac{1}{\kappa_0} \; + \; \mu_{It} && = \; 0 \\
\frac{\partial L}{\partial a_N} \; = \; \frac{\partial L_s(t)}{\partial a_{Nt}} \quad & = \; -\delta_t(\frac{\partial C}{\partial a_N} + \frac{\partial C^{\,S}}{\partial a_N}) \; + \; \lambda_{fNt} \; - \; \lambda_{Bt} \; + \; \mu_{Nt} && = \; 0 \\
\frac{\partial L}{\partial a_U} \; = \; \frac{\partial L_s(t)}{\partial a_{Ut}} \quad & = \; -\frac{\partial C}{\partial a_U}\delta_t \; + \; \lambda_{fUt} \; - \; \lambda_{Bt} \; + \; \mu_{Ut} && = \; 0 \\
\frac{\partial L}{\partial q_{Dt}} \; = \; \frac{\partial L_s(t)}{\partial q_{Dt}} \quad & = \; \frac{\partial B}{\partial q_{Dt}}\delta_t \; + \; \lambda_{Bt} && = \; 0
\end{aligned} \tag{122}
$$

The static optimality conditions considered so far may be combined to show that in each time period the shadow value of the new and used production function constraints ($-\lambda_{fNt}$ and $-\lambda_{fUt}$) are equal to the marginal benefit of output q_D minus the marginal (variable) cost of process activity (capital utilization) a_N or a_U. Thus $-\lambda_{fNt}$ and $-\lambda_{fUt}$ correspond to the discounted marginal value of new and used capital utilization:

$$\delta_t\left(\frac{\partial B_t}{\partial q_{Dt}} - \frac{\partial(C_t + C_t^S)}{\partial a_{Nt}}\right) + \mu_{Nt} = -\lambda_{fNt}$$

$$\delta_t\left(\frac{\partial B_t}{\partial q_{Dt}} - \frac{\partial C_t}{\partial a_{Ut}}\right) + \mu_{Ut} = -\lambda_{fUt} \tag{123}$$

The equations also provide a condition for the efficient level on new capital investment. This condition says that the marginal cost of investment should equal the marginal productivity of investment times the marginal value of investment utilization, plus the shadow value of new capital (since investment is also new capital):

$$\begin{aligned}\frac{\partial C_{It}}{\partial I_t}\delta_t &= -\lambda_{fNt}\frac{\partial f_{Nt}}{\partial I_t} - \lambda_{Kt0} \\ &= -\frac{\lambda_{fNt}}{\kappa_0} - \lambda_{Kt0}\end{aligned} \tag{124}$$

The derivatives of the static Lagrangians with respect to vintaged capital $\boldsymbol{K_{ta}}$ have the same form in each period, and vary with the age of capital $\boldsymbol{a}$. These derivatives are not equal to zero because of the dynamic constraint on capital stock evolution.

$$\begin{aligned}\frac{\partial L_s(t)}{\partial K_{t0}} &= -\frac{\partial C_t}{\partial K_{t0}}\delta_t + \lambda_{Kt0} && a=0,\ \forall t \\ \frac{\partial L_s(t)}{\partial K_{ta}} &= -\frac{\partial C_t}{\partial K_{ta}}\delta_t - \lambda_{fUt}\frac{\partial f_{Ut}}{\partial K_{ta}} \\ &= -\frac{\partial C_t}{\partial K_{ta}}\delta_t - \lambda_{fUt}\frac{1}{\kappa_a} && a>0,\ \forall t\end{aligned} \tag{125}$$

In each period, the derivative of the static Lagrangian with respect to capital stock equals the discounted marginal net use value per unit stock. This marginal use value is given by the marginal net value per unit output ($-\lambda_{Kta}$), times the marginal product of age a capital, $1/\kappa_a$., minus the discounted marginal capital charge ($\partial C/\partial K$), if any.

Now deal with the optimality conditions for $\partial L/\partial K$ for interior (non-boundary) years and for new or old capital

$$\frac{\partial L}{\partial K_{t0}} = \frac{\partial L_s(t)}{\partial K_{t0}} - \lambda_{Kt+1,1}(1-\gamma_0) = 0 \qquad 0<t<T,\ a=0$$

$$\frac{\partial L}{\partial K_{ta}} = \frac{\partial L_s(t)}{\partial K_{ta}} + (\lambda_{Kta} - \lambda_{Kt+1\,a+1}(1-\gamma_a)) = 0 \qquad 0<t<T,\ 0<a<A \tag{126}$$

$$\frac{\partial L}{\partial K_{tA}} = \frac{\partial L_s(t)}{\partial K_{tA}} + \lambda_{KtA} = 0 \qquad 0<t<T,\ a=A$$

Finally, the optimality conditions for ∂L/∂K for special cases at boundary years and for new or old capital

$$\frac{\partial L}{\partial K_{T0}} = \frac{\partial L_s(T)}{\partial K_{T0}} + \delta_T \frac{\partial F_T}{\partial K_{T0}} = 0 \qquad t=T,\ a=0$$

$$\frac{\partial L}{\partial K_{Ta}} = \frac{\partial L_s(T)}{\partial K_{Ta}} + \delta_T \frac{\partial F_T}{\partial K_{Ta}} + \lambda_{KTa} = 0 \qquad t=T,\ 0<a\le A$$

$$\frac{\partial L}{\partial K_{00}} = \frac{\partial L_s(0)}{\partial K_{00}} - \lambda_{K11}(1-\gamma_0) = 0 \qquad t=0,\ a=0 \tag{127}$$

$$\frac{\partial L}{\partial K_{0a}} = \frac{\partial L_s(0)}{\partial K_{0a}} - \lambda_{K1a+1}(1-\gamma_a) + \lambda_{K0a} = 0 \qquad t=0,\ 0<a<A$$

$$\frac{\partial L}{\partial K_{0A}} = \frac{\partial L_s(0)}{\partial K_{0A}} + \lambda_{K0A} = 0 \qquad t=0,\ a=A$$

A2.2.2 Vintaged Stock Shadow Value in Terminal Year

Start with expressions for the shadow value of capital in the terminal period. For new vehicles at time T:

$$\frac{\partial L}{\partial K_{T0}} = -\frac{\partial C_t}{\partial K_{T0}} + \lambda_{KT0} + \delta_T \frac{\partial F_t}{\partial K_{T0}} = 0 \tag{128}$$

Define F_{Ta}, the marginal final value of vintaged stock of age a,

$$\frac{\partial F_T}{\partial K_{Ta}} \equiv F_{Ta} \qquad \forall a \tag{129}$$

In the TAFV model, the marginal final value is specified as a constant, i.e.[19]

$$F_T(\vec{K}) = \sum_{a=0}^{A} K_{Ta} F_{Ta}$$

---

[19]In TAFV, $\boldsymbol{F_{Ta}}$ = FUTUREVV(T,a).

For vehicles, the capital cost is borne at the time of purchase (investment), so the vehicle capital stock does not appear in the cost function:

$$\frac{\partial C_t}{\partial K_{Ta}} = 0 \qquad \forall t,\ a \tag{131}$$

Hence

$$-\lambda_{KT0} = \delta_T F_{T0} \tag{132}$$

Now the shadow value of older vehicles in the terminal period is

$$\begin{aligned} -\lambda_{KTa} &= \delta_T \frac{\partial F_t}{\partial K_{T0}} - \frac{\partial C_t}{\partial K_{T0}} - \frac{\lambda_{fUt}}{\kappa_a} \\ &= \delta_T F_{ta} - \frac{\lambda_{fUt}}{\kappa_a} \qquad a>0 \end{aligned}$$

These conditions are borne out in the numerical results of TAFV.[20]

A2.3 Theoretical Valuation of Used Capital Equipment - An Approach for Scrappage Value

The previous review shows how a dynamic optimization model such as the TAFV model implicitly values capital of age *a* based on its discounted stream of annual use-values for its remaining lifetime. This section derives an estimate of the equilibrium annual use value for vintaged capital in terms of initial capital cost. This annual use value estimate, in turn, is used to construct an estimate of the equilibrium salvage value of vintaged vehicle stock. The estimate of equilibrium salvage value is used in the TAFV model for final valuation of the remaining vehicle stock in the terminal time period. In the fully dynamic model, investments in each period are based on knowledge of capital use-value in the current and all future periods. Our method of final valuation promotes smooth and consistent vintaged-capital investment (vehicle purchase) behavior in the dynamic model, even as the time approaches the terminal period. For the myopic (recursive period-by-period) solution approach, investments in each period are based on current use-value and an expectation of cumulative future value. Our method of vintaged stock valuation is also useful in the myopic solution approach for constructing a myopic estimate of cumulative

---

[20]See SUMMFINV.GMS for a test routine). In terms of the TAFV variable names:

$$\frac{NEWSTKEQ.M(T,0)}{DISCFACT(T)} = FUTUREVV(T,0)$$

$$\frac{STKVEQ.M(T,a)}{DISCFACT(T)} = FUTUREVV(T,a) + \frac{STKVTYPEEQ.M(T)}{STKPERFLOW \cdot DISCFACT(T)} \qquad a>0$$

future vehicle use-value in each period.

A2.3.1 Valuation of New Vehicles

Consider the decision to buy a new piece of capital equipment (e.g. a vehicle) which depreciates over time and whose marginal productivity may decline over time. Define the following terms:

C_{t0}	initial capital cost of new stock at time *t*.
$1/\kappa_a$	productivity of capital of age *a* (services flow per unit stock)
σ_a	survival rate for capital after *a* years (fraction surviving *a* years)
B_t'	marginal net benefit of capital services at time *t* (net of variable costs)
δ_τ	τ-period discount factor, equal to $(\delta)^\tau$ for a constant annual discount factor δ.

Thus the quantity of services provided by the surviving portion of a unit of capital which is *a* years old is σ_a/κ_a. Efficient behavior will lead to purchases of new (age 0) vehicles to the point where the discounted stream of marginal use benefits for a new vehicle ($V(t,0)$) equals the new vehicle cost. Thus for a steady-state or equilibrium outcome we expect:

$$C_{t0} \;=\; V_{t0} \;=\; \sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha} B_{t+\alpha}^{/} \quad \forall t \tag{134}$$

If the new vehicle cost is constant, and if consumers continue to purchase new vehicles, then this relationship will hold for every time *t*. In a stationary model, we would expect marginal undiscounted use-benefits to approach a constant value. In this case, marginal use benefits adjust until:

$$B^{/} \;=\; \frac{C_0}{\displaystyle\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \tag{135}$$

The ratio of the one-period equilibrium marginal use benefit *B'* to initial cost may be interpreted as the "capital charge ratio," r_k, that is the percentage of capital cost that could/should be charged to each unit of use in evaluating whether the investment is worthwhile:

$$r_k \;\equiv\; \frac{B^{/}}{C_0} \;=\; \frac{1}{\displaystyle\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \tag{136}$$

A2.3.2 Valuation of Used Vehicles

Similarly, the shadow value of used capital of age *a* at time *t* should be the discounted value of its remaining use:

$$
\begin{aligned}
V_{ta} &= \sum_{\alpha=a}^{\infty} \delta^{(\alpha-a)} \frac{\sigma_{\alpha|a}}{\kappa_\alpha} B'_{t+\alpha-a} \\
&= \frac{1}{\delta^{-a}\sigma_a} \sum_{\alpha=a}^{\infty} \delta^{\alpha} \frac{\sigma_{\alpha\cap a}}{\kappa_\alpha} B'_{t+\alpha-a}
\end{aligned}
\tag{137}
$$

Here $\sigma_{\alpha|a}$ is the conditional survival rate/probability of surviving to age α given that the capital has survived a years. By definition of conditional probability, $\sigma_{\alpha|a} = \sigma_{a\cap\alpha}/\sigma_a$, where $\sigma_{a\cap\alpha}$ is the joint survival rate/probability of surviving to age a and then to age α for $\alpha \geq a$, i.e. the probability of surviving from age 0 to age α or σ_α. Assuming marginal use-benefit is roughly constant over time, we can omit the time subscript:

$$
V_a = B' \frac{1}{\delta^a \sigma_a} \sum_{\alpha=a}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha} \quad \forall a \geq 0
\tag{138}
$$

Substituting in the expression for "equilibrium" marginal use-benefit B' based on new vehicle cost from Eq. 45, 136, we have an expression for the equilibrium value of used vintaged capital stock in terms of its current age a, initial cost C_0, future survival rates σ_α and future marginal productivity $1/\kappa_\alpha$:

$$
V_a = C_0 \frac{\dfrac{1}{\delta^a \sigma_a} \displaystyle\sum_{\alpha=a}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}}{\displaystyle\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}}
\tag{139}
$$

$$
\frac{V_a}{V_0} = \frac{V_a}{C_0} = \frac{\dfrac{1}{\delta^a \sigma_a} \displaystyle\sum_{\alpha=a}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}}{\displaystyle\sum_{\alpha=0}^{\infty} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}}
\tag{140}
$$

Note that survival rate σ_α and marginal productivity $1/\kappa_\alpha$ (inverse stock-per-flow) appear together and could be combined into a single age-related factor. For vehicles, this means that vehicle scrappage rates and declining use-rates with age are interchangeable, at least in terms of vehicle valuation.[21]

[21] We can identify six factors which affect the value of vintaged capital stock as it ages:

1. discounting, or the opportunity cost of funds (captured by δ^a);
2. scrappage, or the declining fraction of capital surviving (captured by σ_a);
3. declining marginal productivity or use (e.g. miles per vehicle) with age (captured by the marginal productivity factor $1/\kappa_a = \varphi_a/\kappa_0$;
4. the declining *value* of a unit of product/use with the capital's age, e.g. the declining quality of a mile of vehicle services as the vehicle ages;
5. the increasing operating and maintenance costs of capital/vehicles with age;
6. technological obsolescence with the passage of time t and age a, which reduces the value of used capital.

As defined above, the valuation model explicitly accounts for factors 1-3. As a simple approach, factors 4-6 could also be embedded in the declining productivity/use factor φ_a, provided care is exercised to avoid confusing physical units (such as vehicles in the stock and miles traveled) with the units of "value" provided.

A2.2.3 Vintaged Stock Shadow Value in Non-Terminal Years

Starting with the first order conditions from Eq. 127 for non-boundary years and substituting the expressions for $\partial L_s/\partial K$ from Eq.126.

$$
\begin{aligned}
&\lambda_{Kt0} - \lambda_{Kt+1,1}(1-\gamma_0) &&= 0 \qquad 0<t<T,\ a=0\\
&-\frac{\lambda_{fUt}}{\kappa_a} + (\lambda_{Kta} - \lambda_{Kt+1\,a+1}(1-\gamma_a)) &&= 0 \qquad 0<t<T,\ 0<a<A\\
&-\frac{\lambda_{fUt}}{\kappa_A} + \lambda_{KtA} &&= 0 \qquad 0<t<T,\ a=A
\end{aligned}
\tag{141}
$$

Rearranging, we get recursive forms for the shadow value of capital in any period in terms of that period's "use value" (the shadow value of the vehicle services production function constraint for used-vehicles), and next periods shadow value of capital. All of these multipliers represent discounted values. For the given problem statement, the shadow value of capital is given by the negative of the Lagrange multipliers (this is the sensitivity theorem, e.g. Luenberger 1973:236). The GAMS solver reports the shadow values for every constraint, that is it reports the marginal change in the objective function for a unit change in each constraint right-hand-side (Brooke *et al*. 1992:253). Thus the GAMS solver reports the negative of the Lagrange multipliers used in these equations below.

$$
\begin{aligned}
&-\lambda_{Kt0} &&= -\lambda_{Kt+1,1}(1-\gamma_0) &&0<t<T,\ a=0\\
&-\lambda_{Kta} &&= \frac{-\lambda_{fUt}}{\kappa_a} - \lambda_{Kt+1\,a+1}(1-\gamma_a)) &&0<t<T,\ 0<a<A\\
&-\lambda_{KtA} &&= \frac{-\lambda_{fUt}}{\kappa_A} &&0<t<T,\ a=A
\end{aligned}
\tag{142}
$$

Note that the shadow value of new vehicles, $-\lambda_{Kt0}$, depends only on next year's shadow value of one-year old vehicles. This is because the use of new vehicles is attributed to investment $\boldsymbol{I_t}$ in Eq. 119 rather than new capital K_{t0}.

The shadow value of capital can be written in closed (summation) form, rather than recursive form:

$$
\begin{aligned}
&-\lambda_{Kt0} &&= \sum_{\alpha=1}^{A}\left(\frac{-\lambda_{fU(t+\alpha)}}{\kappa_a}\prod_{\sigma=1}^{\alpha}(1-\gamma_{\sigma-1})\right) &&0<t<T-A,\ a=0\\
&-\lambda_{Kta} &&= \frac{-\lambda_{fUt}}{\kappa_a} + \sum_{\alpha=a+1}^{A}\left(\frac{-\lambda_{fU(t+(\alpha-a))}}{\kappa_\alpha}\prod_{\sigma=a+1}^{\alpha}(1-\gamma_{\sigma-1})\right) &&0<t<T-(A-a),\ 0<a<A\\
&-\lambda_{KtA} &&= \frac{-\lambda_{fUt}}{\kappa_A} &&0<t<T,\ a=A
\end{aligned}
\tag{143}
$$

Another way to write these equations is in terms of the *survival* rates σ_a, where σ_a is the fraction of stock surviving to age *a* (i.e., not being scrapped in any period prior to *a*):

$$\sigma_a \equiv \begin{cases} \prod_{\alpha=0}^{a-1}(1-\gamma_\alpha) = \prod_{\alpha=1}^{a}(1-\gamma_{\alpha-1}) & a>0 \\ 1 & a=0 \end{cases} \tag{144}$$

The conditional survival rate for surviving to age α given that the capital has survived to age ***a*** has the simple form:

$$\Rightarrow \prod_{m=a+1}^{\alpha}(1-\gamma_{m-1}) = \frac{\sigma_\alpha}{\sigma_a} \tag{145}$$

Using these survival rates, the equations are simpler, as is the treatment of special cases for age *a*:

$$\begin{aligned} -\lambda_{Kt0} &= \sum_{\alpha=1}^{A}\left(\frac{-\lambda_{fU(t+\alpha)}}{\kappa_a}\sigma_a\right) && 0<t<T-A,\ a=0 \\ -\lambda_{Kta} &= \sum_{\alpha=a}^{A}\left(\frac{-\lambda_{fU(t+(\alpha-a))}}{\kappa_\alpha}\frac{\sigma_\alpha}{\sigma_a}\right) && 0<t<T-(A-a),\ 0<a\leq A \end{aligned} \tag{146}$$

The equation for the shadow value of age zero capital differs from that for older vintages because, as indicated above, the use value of new capital is attributed to investment shadow price multiplier. This essentially arbitrary assignment was convenient, and it does not change the solution since investment equals new capital.

These equations apply *so long as* we don't encounter the terminal time point T before the vehicle is scrapped, i.e., so long as the time is less than *A-a* periods from the end of the time horizon. Otherwise, future marginal use values are replaced by the exogenous marginal scrappage or final value F_{Ta}.

$$\begin{aligned} -\lambda_{Kt0} &= \sum_{\alpha=1}^{T-t}\left(\frac{-\lambda_{fU(t+\alpha)}}{\kappa_\alpha}\sigma_\alpha\right) + \delta^T F_{T(T-t)}\sigma_{(T-t)} && T-(A-a)<t\leq T,\ a=0 \\ -\lambda_{Kta} &= \sum_{\alpha=a}^{T-t+a}\left(\frac{-\lambda_{fU(t+(\alpha-a))}}{\kappa_\alpha}\frac{\sigma_\alpha}{\sigma_a}\right) + \delta^T F_{T(a+T-t)}\frac{\sigma_{(a+T-t)}}{\sigma_a} && T-(A-a)<t\leq T,\ 0<a\leq A \end{aligned} \tag{147}$$

The marginal productivity of capital, $1/\kappa_\alpha$, corresponds to the number of miles of vehicle services provided by a vehicle of age α. This allows us to account for declining vehicle use with age. Even more generally, marginal productivity could also change with time, and $1/\kappa_\alpha$ could be replaced by the marginal productivity of age α capital α-a years in the future:

$1/\kappa_{t+\alpha-a,\alpha} = \partial f_{U(t+\alpha-a)}/\partial K_{(t+\alpha-a)\alpha}$.

$1/\kappa_{t0} = \partial f_{Nt}/\partial I_t$.

In terms of vehicle stock, this would correspond to an exogenous trend in the number of miles driven by vehicles of each age. This refinement was not deemed necessary for the TAFV model. The final scrappage value could be given one more period of discounting and the stock aged by one more year, to account for year-end scrappage after terminal year use. As an alternative approach, the extra discounting and aging are built into the final valuation function F_{Ta}.

A2.4 Used Vehicle Valuation for Particular Scrappage Profiles

A2.4.1 Infinite-lived, Constant Depreciation Rate Case

Consider first the case of infinite-lived, but geometrically depreciating capital, with constant depreciation rate γ, and constant marginal productivity of surviving capital (κ_a=κ). In this case, the survival probability is Poission, i.e. $\sigma_a = \sigma^a$ and $\sigma_{\alpha|a} = \sigma^{\alpha\text{-}a}$. The value of capital of age a simplifies:

$$V(a) = C_0 \frac{(\delta\sigma)^{-a}\sum_{\alpha=a}^{\infty}\delta^\alpha\sigma^\alpha}{\sum_{\alpha=0}^{\infty}\delta^\alpha\sigma^\alpha} = \frac{C_0}{(\delta\sigma)^a}\frac{\sum_{\alpha=0}^{\infty}\delta^\alpha\sigma^\alpha - \sum_{\alpha=0}^{a-1}\delta^\alpha\sigma^\alpha}{\sum_{\alpha=0}^{\infty}\delta^\alpha\sigma^\alpha} = \frac{C_0}{(\delta\sigma)^a}\frac{\frac{1}{1-\delta\sigma} - \frac{1-(\delta\sigma)^a}{1-\delta\sigma}}{\frac{1}{1-\delta\sigma}} \tag{148}$$

$$\Rightarrow V(a) = C_0, \qquad \text{where} \quad \delta = \frac{1}{1+r}, \quad \sigma = 1-\gamma$$

Thus the value of used infinite-lived, geometrically depreciating capital, conditional on the fact that it has survived, is equal to the value of new capital. In this case the capital is essentially unvintaged. This simple result depends on the assumption of constant productivity/usefulness with age.

A2.4.2 Finite-lifetime, Zero Depreciation Until Final Age Case

Now suppose that the capital has a finite lifetime A with no depreciation prior to age A, (σ_a=1 for $a \le A$) and that marginal capital productivity of surviving capital is constant, i.e. stock-per-flow κ_a is constant with age.

$$V_a = C_0 \frac{\delta^{-a}\sum_{\alpha=a}^{A}\delta^\alpha\frac{\sigma_\alpha}{\kappa_\alpha}}{\sum_{\alpha=0}^{A}\delta^\alpha\frac{\sigma_a}{\kappa_a}} = C_0\delta^{-a}\frac{\sum_{\alpha=a}^{A}\delta^\alpha}{\sum_{\alpha=0}^{A}\delta^\alpha} = C_0\delta^{-a}\frac{\sum_{\alpha=0}^{A}\delta^\alpha - \sum_{\alpha=0}^{a-1}\delta^\alpha}{\sum_{\alpha=0}^{A}\delta^\alpha} \tag{149}$$

$$= C_0\delta^{-a}\frac{\frac{1-\delta^{A+1}}{1-\delta} - \frac{1-\delta^a}{1-\delta}}{\frac{1-\delta^{A+1}}{1-\delta}} = C_0\delta^{-a}\frac{\delta^a - \delta^{A+1}}{1-\delta^{A+1}}$$

For the somewhat more general combined case of finite lifetime A, but geometrically depreciating capital, the above equation reduces to the following expression.

$$V(a) \ = \ C_0 \left[\frac{1 \ - \ (\delta\sigma)^{A+1-a}}{1 \ - \ (\delta\sigma)^{A+1}} \right]$$

A2.4.3 Properties of this Result for Finite Lifetime No Depreciation Case

First note that this approximation assumes vehicle cost and marginal use benefit is constant, and thus is a steady-state equilibrium approximation. The value calculated by this result for a vehicle

$$V_a \ = \ C_0 \frac{1-\delta^{A+1-a}}{1-\delta^{A+1}} \tag{150}$$

in its last year is equal to the presumed equilibrium value of one-year's use-benefits:

$$\begin{aligned} V_A \ &= \ C_0 \frac{1-\delta\sigma}{1-(\delta\sigma)^{A+1}} \\ &= \ \frac{C_0}{\sum_{\alpha=0}^{A} (\delta\sigma)^{\alpha}} \ = \ \frac{B^{/}}{\kappa} \end{aligned}$$

One disturbing result is that V_a declines with a, but at an increasing rate. For example, the value of a vehicle with one year left in its life (V_A) exceeds the difference in value between a new and a one-year old vehicle:

$$V_A > (V_0 - V_1).$$

In fact, we can show that for constant marginal productivity of capital ($1/\kappa$) the difference in values for two vehicles of age a and a+1 is just one-year's equilibrium use-benefit $B'/\kappa = V_A$ discounted and depreciated by A-a periods:

$$V_a - V_{a+1} = (\delta\sigma)^{A-a} \, V_A.$$

The smallest difference is thus between the values of a brand new vehicle and a vehicle of age 1:

$$V_0 - V_1 = (\delta\sigma)^{A} \, V_A.$$

This is the opposite of the pattern of value-depreciation which we observe empirically (where the loss in value is fastest in the early years). The discrepancy occurs probably because the marginal productivity of capital is <u>not</u> constant with age.

In the more general case of age-dependent marginal productivity κ_a, the change in value for one year of aging is:

$$V_a - V_{a+1} \ = \ \frac{C_0}{\sum_{\alpha=0}^{A} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \left[\frac{1}{\delta^{a}\sigma_a} \sum_{\alpha=a}^{A} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha} \ - \ \frac{1}{\delta^{(a+1)}\sigma_{a+1}} \sum_{\alpha=a+1}^{A} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha} \right] \tag{153}$$

Using our estimated steady-state equilibrium relationship between marginal use benefit B' and initial capital cost $C_{0,}$ we can construct an expression for the loss in value of vintaged capital stock with age:

$$B^{/} = \frac{C_0}{\sum_{\alpha=0}^{A} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha}} \tag{154}$$

$$V_A = \frac{B^{/}}{\delta^A \sigma_A} \sum_{\alpha=A}^{A} \delta^{\alpha} \frac{\sigma_\alpha}{\kappa_\alpha} = \frac{B^{/}}{\kappa_A}$$

$$V_a - V_{a+1} = V_A \kappa_A \left[\frac{1}{(\delta\sigma)^a} \sum_{\alpha=a}^{A} \frac{(\delta\sigma)^\alpha}{\kappa_\alpha} - \frac{1}{(\delta\sigma)^{(a+1)}} \sum_{\alpha=a+1}^{A} \frac{(\delta\sigma)^\alpha}{\kappa_\alpha} \right] \tag{155}$$

This more general form yields the same result as above geometric discounting and scrappage case for constant κ with age.

$$\begin{aligned} V_a - V_{a+1} &= \frac{V_A}{(\delta\sigma)^a} \left[\sum_{\alpha=a}^{A} (\delta\sigma)^\alpha - \frac{1}{(\delta\sigma)} \sum_{\alpha=a+1}^{A} (\delta\sigma)^\alpha \right] \\ &= \frac{V_A}{(\delta\sigma)^a} \left[\sum_{\alpha=a}^{A} (\delta\sigma)^\alpha - \sum_{\alpha=a}^{A-1} (\delta\sigma)^\alpha \right] \\ &= V_A (\delta\sigma)^{A-a} \end{aligned} \tag{156}$$

Otherwise, for capital productivity/use κ_a varying with age, the result is not easily simplified.

A2.4.2 Valuation of Used Vintaged Capital Based on Theoretical Equilibrium Use-Value, General Case

We return to consideration of the general case of vintage capital with an arbitrary scrappage rate, use rate, and lifetime. Define ρ_a to be the effective remaining productivity of capital, *conditional* on it having survived to age *a*:

$$\rho_a \equiv \frac{1}{\delta^a \sigma_a} \sum_{\alpha=a}^{\infty} \frac{\delta^\alpha \sigma_\alpha}{\kappa_\alpha} \tag{157}$$

For any scrappage profile σ_α and productivity/use profile $1/\kappa_\alpha$, these coefficients can be calculated for an arbitrary number of ages. Then the effective remaining fraction of original use value is ρ(a)/ρ(0), and from Equation 46, 140:

$$V(a) = C_0 \frac{\rho(a)}{\rho(0)}$$

We base the final value function F_{Ta} per unit capital of age *a* on this salvage value estimate V_a, recognizing that since salvage occurs at the *end* of the terminal period, the capital must survive to be one period older and the salvage value must be discounted by one more period.

The undiscounted salvage value per unit of capital at age *a* remaining at (the start of) terminal time T:

$$F_{Ta} = \delta V_{a+1}\frac{\sigma_{a+1}}{\sigma_a}$$
$$= \delta C_0 \frac{\rho_{a+1}}{\rho_0}\frac{\sigma_{a+1}}{\sigma_a}$$

Given the revised model formulation for vintaged stock in terms of historical investments alone, we are also interested in the NPV of salvage value per unit of new investment at time t $\leq$ T (for FINALVAL equation in V61+). Since investment in time t (I_t) yields capital of age (T-t) at terminal time:

$$K_{T,T-t} = \sigma_{T-t} I_t$$

The most recent version of the TAFV model tracks investment in all periods, avoiding the explicit tracking of vintaged capital stocks (see Appendix 3). We can evaluate the final valuation of period-t investment (I_t) in terms of the above equation for the final valuation of surviving capital and the survival probability from time t to T:

$$\frac{\partial F_T}{\partial I_t} = \frac{\partial F_T}{\partial K_{T,T-t}}\frac{\partial K_{T,T-t}}{\partial I_t} = \frac{\partial F_T}{\partial K_{T,T-t}}\sigma_{T-t}$$
$$= \delta V_{T-t+1}\left[\frac{\sigma_{T-t+1}}{\sigma_{T-t}}\right]\sigma_{T-t}$$
$$= \delta V_{T-t+1}\sigma_{T-t+1}$$
$$= F_{T,T-t}\sigma_{T-t}$$

From these results we can see that the discounted unit salvage value of capital of age *a* at time *t* should exceed the unit salvage value of new investment at time *t-a*, since survival to age *a* is not assured:

$$\frac{\partial F_T}{\partial K_{ta}}\frac{\sigma_a}{\sigma_0} = \frac{\partial F_T}{\partial I_{t-a}}$$
$$= \delta V_{a+T-t+1}\frac{\sigma_{a+T-t+1}}{\sigma_a}\frac{\sigma_a}{\sigma_0}$$
$$= \delta V_{T-(t-a)+1}\sigma_{T-(t-a)+1}$$

Similarly, the prospective discounted unit salvage value of capital of age *a* at time t is less than the unit salvage value of capital of age *a*+(*T-t*) at time *T*:

$$
\begin{aligned}
\frac{\partial F_T}{\partial K_{ta}} \frac{\sigma_{a+(T-t)}}{\sigma_a} &= \frac{\partial F_T}{\partial K_{T,a+T-t}} \\
&= \delta V_{a+T-t+1} \frac{\sigma_{a+T-t+1}}{\sigma_{a+T-t}} \frac{\sigma_{a+T-t}}{\sigma_a} \\
&= \delta V_{a+T-t+1} \frac{\sigma_{a+T-t+1}}{\sigma_a}
\end{aligned}
$$

A2.5 Steady-State Method of Dealing with Boundary Condition in Ramsey Model

This appendix has examined methods for setting a terminal “salvage” value for capital stock, in order to promote rational model investment behavior in the periods leading up to the terminal one. Alternatively, the terminal time problem can be handled by modifying the terminal period benefits and constraints to reflect steady-state conditions which are assumed to persist indefinitely after the terminal period. We review the approach used by Manne [1986] in his GAMS implementation of the Ramsey model (Ramsey, 1928).

The Ramsey (1928) model is a classical exploration of the tradeoff between consumption and capital investment over time. It maximizes the net present value of consumption utility, allocating output Y between investment I and consumption C in each period:

$$
\begin{aligned}
Y &= a_t K_t^{\,b} \\
Y_t &= C_t + I_t \\
K_{t+1} &= K_t + I_t \\
gK_T &\le I_T
\end{aligned}
$$

The single-period utility is logarithmic in consumption C. The multiperiod utility is the discounted sum of single period utilities.

$$U = \sum_{t=0}^{T} \beta(t)\ln(C_t)$$

$$\beta(t) = \begin{cases} \delta^t & 0 \le t < T \\ \dfrac{\delta^T}{(1-\delta)} & t = T \end{cases}$$

This finite-horizon dynamic model uses two mechanisms to promote reasonable behavior in the terminal time period T:

1. It requires a minimum level of investment in the terminal period ($I_T \ge gK_T$).
2. It weights final period net consumption benefits more heavily.

The first mechanism requires that terminal period investment be sufficient to account for growth in the labor supply, $\boldsymbol{g}$. The growth of labor supply is also reflected in the scale coefficient for the production function, which is benchmarked as follows:

$$a = \frac{C_0 + I_0}{Y_0}$$

$$a_t = a(1+g)^{t(1-B)}$$

The new investment in terminal period T is required to be sufficient to accommodate expected population (labor supply) growth g. In this model, there is no capital depreciation. For our model, last period investment should be sufficient to account for both growing demand and capital depreciation

The second mechanism simply weights the final period benefits more heavily, corresponding to the presumption that those benefits will be maintained as a steady state outcome. The multiplicative weight $1/(1-\delta)$ applied to the terminal period discount factor corresponds to the net present value of an perpetuity discounted with the discount factor δ. The combined discount factor $\delta^T/(1-\delta)$ is then the NPV at time t=0 of a perpetuity beginning at time ***T***:

$$\delta = \frac{1}{1+r}$$

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta} = \frac{1+r}{r}$$

Note that this approach weights the terminal period "intermediate value" benefits function (i.e., the terminal period capital stock *use* benefits) rather than assigning value to the capital terminal stock. There is no terminal stock valuation function. This approach provides an alternative to the calculation and use of terminal stock salvage values. Except for the constraint on terminal period investment, it is conceptually similar to using terminal stock value equations such as those we present above. Rather than calculating the NPV of the remaining marginal product stream for each vintage of capital, it assumes that the terminal period marginal benefit will persist, along with the capital. To apply this method to the TAFV model, two adjustments may be required. The scrappage rate for capital must be recognized when calculating the final-period discount rate. It also must be recognized that some of the net benefits in the terminal period are not attributable to the capital stock, and thus perhaps should not be included in the steady-state perpetuity calculation. If the fraction γ of capital is scrapped each year, then the annual survival rate is $\sigma = 1-\gamma$. This is combined with the discount factor δ to produce the terminal period steady-state discount factor $\beta(T)$:

$$\beta(T) = \frac{(\delta\sigma)^T}{(1-\delta\sigma)}$$

The current version of TAFV model does not use this steady-state approach to correcting for the finite time horizon, but it may prove a useful alternative.

APPENDIX 3: REDUCING THE NUMBER OF MODEL EQUATIONS

We observed that many of the variables in earlier versions of the model (as of TAFV-V56) were due to the vintaging of vehicle stock. Although the equations for vintaged vehicle stock are indeed simple, they do add substantially to the model size, making the solution of the full model problematic even with machines which have 64 megabytes of memory. Apparently, in GAMS even simple variables can, by increasing the problem matrix dimension, require over 4K of memory each.

We reduced the number of equations and variables in the model by a simple problem restatement. The problem is mathematically unchanged, and no more easy to solve in terms of complexity, but now requires substantially less memory.

A3.1 Explicit Tracking of Vintages by Variables

Consider first the use of model constraints related to the tracking and use of new and used vintaged vehicle stock (omitting non-negativity constraints):[22]

Variable Name:	*Equation*:	*L−mult Symbol*:	*Condition*:
STKVEQ:	$K_{t+1\ a+1} = K_{ta}(1-\gamma_a)$	$\lambda_{Kt+1a+1}$	$t=0......T-1,\ a=0......A-1$
NEWSTKEQ:	$K_{t0} = I_t$	λ_{Kt0}	$\forall t$
STKVTYPEEQ:	$a_{Ut} \le \sum_{a=1}^{A} \frac{K_{ta}}{\kappa_a} \equiv f_U(\vec{K})$	λ_{fUt}	$\forall t$
NEWUSEEQ:	$a_{Nt} = \frac{I_t}{\kappa_0} \equiv f_N(I_t)$	λ_{fNt}	$\forall t$
STKPCTE0:	$K_{0a} \ge \alpha_a K_0$	λ_{KUa}	$a>0,\ t=0$

For each vehicle type, this representation has the following number of equations:

Equation	Number
STKVEQ	$(T-1)(A-1)$
NEWSTKEQ,STKVTYPEQ,NEWUSEEQ	$3\ T$
STKPCTE0	$A-1$
Total	$2T + AT$

For each vehicle type, this representation has the following number of variables:

Variable	Number
K_{ta}	$T\ A$
I_t, a_{Ut}, a_{Nt}	$3T$
Total	$3T + AT$

[22]This method was used in TAFV versions (TAFV-V52-TAFV-V56).

For 30 years and 10 vintages (ages), this implies 360 equations and 390 variables per vehicle. Since vehicles routinely last longer than 10 years, longer vintaging, and proportionally more variables, will be required if the vintaged capital stock is explicitly tracked. For 30 years and 30 vintages, this approach implies 960 equations and 990 variables per vehicle.

Note that the vintaged vehicle stocks K_{ta} are not used for much, they are just used to track stock aging and retirement, and all used stock is summed up in the used-vehicle usage constraint, STKVTYPEEQ. This suggests an opportunity for simplification.

A3.2 Alternative Methodology

Now consider an alternative approach, where we do not track the vintaged vehicle stock explicitly, but rather track new vehicle investment in each year, I_t. At any later time, it is easy to calculate the implied number of surviving vehicles, which is the same as the vintaged vehicle stock. This is possible because vehicle scrappage rates are exogenous.

Rather than deal in single-period age-based scrappage rates γ_a, it is more convenient to use cumulative survival rates for age a, σ_a. Prior to solution, we calculate the survival rate parameter σ_a, which denotes the probability of surviving *to* age a, i.e., the fraction not scrapped prior to age a:

$$\sigma_a = \begin{cases} \prod_{\alpha=0}^{a-1} (1-\gamma_\alpha) & a>1 \\ 1 & a=0 \end{cases}$$

We need pre-calculated survival rates for ages up to $T+A$ (where T is the length of the time horizon and A is the oldest historical vehicle age of interest at time t=0). If vehicles are finite-lived with lifetime A, set $\sigma_a = 0$ for $a > A$.

To treat all new vehicle investments identically, we added A historical periods to the current T projected time periods. Thus, we now assume that time t runs from $-A \leq t \leq T$, rather than $0 \leq t \leq T$. However, all events prior to t=0 are predetermined, so no new equations are added. Fixed values for the investment variable (vehicle purchases) are added for A historical periods. We can reconstruct the historical path of new vehicle purchases given the exogenous starting stock by age and the survival rates. For all periods prior to the first forecast year in t=0:

$$I_{0-a} = \frac{K_{0a}}{\sigma_a} = \frac{\alpha_a K_0}{\sigma_a} \qquad a=1...A$$

Given the survival rates and a complete path of historical and projected vehicle investments, we can calculate (and substitute for) vintaged vehicle stock "on the fly", rather than track it explicitly:

$$K_{ta} \equiv I_{t-a}\sigma_a$$

Thus we raise the vehicle stock utilization constraint (*STKVTYPEEQ*):

$$a_{ut} \le \sum_{a=1}^{A} \frac{I_{t-a}\sigma_a}{\kappa_a} = \sum_{\tau=0-A}^{t-1} \frac{I_\tau \sigma_{t-\tau}}{\kappa_{t-\tau}}$$

In the right-most expression we see why constructing survival rates for very large vehicle ages might be handy. An even simpler form is possible if we precalculate what we call the "meta-survival" coefficients $\varphi_{t\tau}$, which are the effective fractions of used vehicles bought in time τ which are available for use at time t. They account for both scrappage and declining use or productivity with age. "Meta-survival" coefficients $\varphi_{t\tau}$ may be defined for all t, τ, but are zero for $t < \tau$ (vehicles which are purchased this year or in future years cannot contribute to current used vehicle stock):

$$\varphi_{t\tau} = \begin{cases} \sigma_{t-\tau}\dfrac{\kappa_0}{\kappa_{t-\tau}} & \textit{for } \tau < t \\ 0 & \textit{for } \tau \ge t \end{cases}$$

$$a_{Ut} \le \sum_{\tau=-A}^{T} \frac{I_\tau \varphi_{t\tau}}{\kappa_0}$$

With this preparation, the new model equations related to vehicle purchases and the use of new and used (implicitly) vintaged vehicle stock are:

Variable Name:	*Equation*:	*L-mult Symbol*:	*Condition*:
STKVTYPEEQ:	$a_{Ut} \le \sum_{\tau=-A}^{t-1} \frac{I_\tau \sigma_{t-\tau}}{\kappa_{t-\tau}} \equiv f_U(\vec{K})$	λ_{fUt}	$\forall t \ge 0$
or:	$a_{Ut} \le \sum_{\tau=-A}^{T} \frac{I_\tau \varphi_{t\tau}}{\kappa_0} \equiv f_U(\vec{K})$	λ_{fUt}	$\forall t \ge 0$
NEWUSEEQ:	$a_{Nt} = \frac{I_t}{\kappa_0} \equiv f_N(I_t)$	λ_{fNt}	$\forall t \ge 0$

For each vehicle type, this representation has the following number of equations:

Equation	Number
STKVTYPEQ,*NEWUSEEQ*	$2T$
Total	$2T$

For each vehicle type, this representation has the following number of variables:

Variable	Number
I_t, a_{Ut}, a_{Nt}	$(T+A) + 2T$
Total	$3T+A$ (of which, A exogenous)

For a 30 year time projection and 10+ vintages (ages), this implies 60 equations and 90 endogenous variables per vehicle, plus A=10 fixed historical investment variables. For 30 year time horizon and 30 vintages this approach still requires only 60 equations and 90 endogenous variables per vehicle.

This restatement was adopted for the current version of the TAFV model. It dramatically reduced the number of variables and eliminated the usual dynamic state equation (STKVEQ). All of the dynamics of vehicle stock are embedded in the used-vehicle usage constraint (STKTYPEEQ). This is now a somewhat more complicated equation, because is says that investment in *any* period *could* be relevant to the current use constraint. However, the meta-survival coefficients $\varphi_{t\tau}$ limit which year's investments matter, and the solution of the revised problem is identical to the original one.[23]

The version of the vintaged stock utilization constraint (STKVTYPEEQ) in the new approach that relies on the "meta-survival" coefficients $\varphi_{t\tau}$ is easier to implement, since it involves a sum over all *t* in the GAMS equation. This is less tricky than a sum over selected *t* on the right-hand-side of a GAMS equation.

An important feature of this new approach is that we can allow an arbitrarily long vehicle life (e.g. unlimited statistical life) with *no* additional endogenous variables in the equation. Also, it remains compatible with declining usage with age (indicated by the subscripting of κ).

[23] In fact, if GAMS was very clever, it might have already made many of these substitutions, and this change would have no effect. This is apparently not the case, however, since the proliferation of variables due to vintaging seems to affect the reported model size and its ability to solve within available memory.